Math 1090 ~ Business Algebra

Section 2.5 Application Problems with Matrices

Objectives:
• Employ a variety of strategies to solve systems of equations.
• Examine an example of matrices as used in encryption.
Application Problems with Matrices

Ex 1: (Encryption)

Use \( M = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 3 & -2 & 2 \end{bmatrix} \) to encrypt "JOYFUL" where \( A = 1, B = 2, \) etc.

\( A = 1, B = 2, C = 3, D = 4, \ldots, Y = 25, Z = 26 \)

\( J = 10, O = 15, Y = 25, F = 6, U = 21, L = 12 \)

JOYFUL becomes \( \begin{bmatrix} 10 \\ 15 \\ 25 \end{bmatrix} \) and \( \begin{bmatrix} 6 \\ 21 \\ 12 \end{bmatrix} \)

Encode the message:

\[
\begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 10 \\ 15 \\ 25 \end{bmatrix} = \begin{bmatrix} 10-30+75 \\ -40+75-150 \\ 30-30+50 \end{bmatrix} = \begin{bmatrix} 55 \\ -115 \\ 50 \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 21 \\ 12 \end{bmatrix} = \begin{bmatrix} 6-42+36 \\ -24+105-72 \\ 18-42+24 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ 0 \end{bmatrix}
\]

\( \Rightarrow \) encoded message: \( \begin{bmatrix} 55 \\ -115 \\ 50 \end{bmatrix} \)

The receiver of the encoded message would decode the message by multiplying by

\( M^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 2 & 3 \\ 10 & 7 & 6 \\ 7 & 4 & 3 \end{bmatrix} \)
Ex 2: A grocer is going to mix three kinds of nuts to make 40 lb. of a mixture that will be priced at $5.95/lb. The three kinds of nuts are peanuts priced at $4.00/lb., cashews at $6.60/lb., and pistachios at $8.20/lb. The mixture will contain twice as much in peanuts as cashews by weight. How many pounds of each nut are in the mix?

<table>
<thead>
<tr>
<th></th>
<th>wt</th>
<th>cost/lb</th>
<th>total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>peanut</td>
<td>x</td>
<td>4.00</td>
<td>4x</td>
</tr>
<tr>
<td>cashews</td>
<td>y</td>
<td>6.60</td>
<td>6.60y</td>
</tr>
<tr>
<td>pistachios</td>
<td>2</td>
<td>8.20</td>
<td>8.20x y</td>
</tr>
<tr>
<td>mix</td>
<td>40</td>
<td>5.95</td>
<td>59.5(40)</td>
</tr>
</tbody>
</table>

1. \(4x + 6.60y + 8.20z = 59.5\) (40)
2. \(x + y + z = 40\)
3. \(x = 2y\) \(\Rightarrow\) \(x - 2y = 0\)

Note: 1 is same as
\(40x + 66y + 82z = 59.5\) (40)
\(\Rightarrow\) \(40x + 66y + 82z = 2380\)

We'll solve it using
\[AX = B\]
\[A = \begin{bmatrix} 40 & 6.6 & 82 \\ 1 & 1 & 1 \\ 1 & -2 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 2380 \\ 40 \end{bmatrix}\]

\[A^{-1} = \frac{1}{100} \begin{bmatrix} 2 & 16.4 & 16 \\ -1 & 82 & -42 \\ 3 & -146 & 26 \end{bmatrix}\]

\[X = A^{-1}B = \frac{1}{100} \begin{bmatrix} 2 & 16.4 & 16 \\ -1 & 82 & -42 \\ 3 & -146 & 26 \end{bmatrix} \begin{bmatrix} 2380 \\ 40 \end{bmatrix} = \begin{bmatrix} 18 \\ 9 \\ 13 \end{bmatrix} \Rightarrow \text{in mixture, we want}\]

\[18 \text{ lb peanuts}, \quad 9 \text{ lb cashews}, \quad 13 \text{ lb pistachios}\]
Ex 3: A company needs to borrow $150,000. For tax and related reasons, the company wants to pay 7.3% interest on this loan. There are three lenders for this money. The first charges 6%, the second charges 7% and the third charges 10%. The company is going to borrow twice as much from the first lender as from the third. How much should the company borrow from each lender?

\[
\begin{array}{c|c|c}
\text{No.} & \text{Amount} & \text{Rate} \\
\hline
1 & x & 0.06x \\
2 & y & 0.07y \\
3 & z & 0.10z \\
\hline
\end{array}
\]

\[
\begin{align*}
\text{A} & \quad x + y + 2z = 150,000 \\
\text{B} & \quad 0.06x + 0.07y + 0.10z = 150,000 (0.073) \\
\text{C} & \quad x = 2z \\
\Rightarrow & \quad x - 2z = 0
\end{align*}
\]

Using substitution, \(x = 2z\), we get

\[
\begin{align*}
\text{A} & \quad 2z + y + 2z = 150,000 \\
\text{B} & \quad 6z + 7y + 10z = 1,095,000
\end{align*}
\]

\[
\begin{align*}
\text{A} & \quad y + 3z = 150,000 \\
\text{B} & \quad 7y + 22z = 1,095,000
\end{align*}
\]

\[
\begin{align*}
\text{(7)} & \quad 1 \quad 3 : 150,000 \\
\text{(7)} & \quad 7 \quad 22 : 1,095,000
\end{align*}
\]

\[
\begin{array}{c|c|c}
\text{Row} & \text{Column} & \text{Value} \\
\hline
1 & 3 & 150,000 \\
0 & 1 & 45,000
\end{array}
\]

\[
\begin{align*}
y & = 15,000 \\
z & = 45,000 \\
x & = 90,000
\end{align*}
\]