Math 1090 ~ Business Algebra

Section 2.5 Application Problems with Matrices

Objectives:
• Employ a variety of strategies to solve systems of equations.
• Examine an example of matrices as used in encryption.

\[ 5x - 2y \leq 75 \]

\[
\begin{bmatrix} a & b \\ c & d \end{bmatrix}
\]

\[ S = Pe^r \]

\[ APY = \left(1 + \frac{r}{n}\right)^n - 1 \]
Application Problems with Matrices

Ex 1: (Encryption)

Use $M = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 3 & -2 & 2 \end{bmatrix}$ to encrypt "JOYFUL" where A = 1, B = 2, etc.

$A=1, B=2, C=3, D=4, ..., Y=25, Z=26$

$J=10, O=15, Y=25, F=6, U=21, L=12$

JOYFUL becomes $\begin{bmatrix} 10 \\ 15 \\ 25 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 21 \\ 12 \end{bmatrix}$

**Encode the message:**

$\begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 10 \\ 15 \\ 25 \end{bmatrix} = \begin{bmatrix} 10-30+75 \\ -40+75-150 \\ 30-30+50 \end{bmatrix} = \begin{bmatrix} 55 \\ -115 \\ 50 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 21 \\ 12 \end{bmatrix} = \begin{bmatrix} 6-42+36 \\ -24+105-72 \\ 18-42+24 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ 0 \end{bmatrix}$

$\Rightarrow$ encoded message: $\begin{bmatrix} 55 \\ -115 \\ 50 \end{bmatrix}$

The receiver of the encoded message would decode the message by multiplying by

$M^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 2 & 3 \\ 10 & 7 & 6 \\ 7 & 4 & 3 \end{bmatrix}$
Ex 2: A grocer is going to mix three kinds of nuts to make 40 lb. of a mixture that will be priced at $5.95/lb. The three kinds of nuts are peanuts priced at $4.00/lb., cashews at $6.60/lb., and pistachios at $8.20/lb. The mixture will contain twice as much in peanuts as cashews by weight. How many pounds of each nut are in the mix?

<table>
<thead>
<tr>
<th></th>
<th>weight (lb)</th>
<th>cost per lb</th>
<th>total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>peanuts</td>
<td>x</td>
<td>4.00</td>
<td>4x</td>
</tr>
<tr>
<td>cashews</td>
<td>y</td>
<td>6.60</td>
<td>6.60y</td>
</tr>
<tr>
<td>pistachios</td>
<td>0.5</td>
<td>8.20</td>
<td>4.10y</td>
</tr>
<tr>
<td>mix</td>
<td>40</td>
<td>5.95</td>
<td>5.95(40)</td>
</tr>
</tbody>
</table>

1. \(4x + 6.60y + 8.20z = 5.95(40)\)
2. \(x + y + z = 40\)
3. \(x = 2y \implies x - 2y = 0\)

Note: 1 is same as
\[40x + 66y + 82z = 595(40)\]
\[\iff 40x + 66y + 82z = 2380\]

We'll solve it using
\[AX = B\]

\[
A = \begin{bmatrix}
40 & 64 & 82 \\
1 & 1 & 1 \\
1 & -2 & 0
\end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 2380 \\ 40 \\ 0 \end{bmatrix}
\]

\[
A^{-1} = \frac{1}{100} \begin{bmatrix}
-2 & 164 & 16 \\
-1 & 82 & -42 \\
3 & -146 & 26
\end{bmatrix}
\]

\[
X = A^{-1}B = \frac{1}{100} \begin{bmatrix}
-2 & 164 & 16 \\
-1 & 82 & -42 \\
3 & -146 & 26
\end{bmatrix} \begin{bmatrix} 2380 \\ 40 \\ 0 \end{bmatrix} = \begin{bmatrix} 18 \\ 9 \\ 13 \end{bmatrix}
\]

\(\Rightarrow\) in mixture, we want

- 18 lb peanuts
- 9 lb cashews
- 13 lb pistachios
Ex 3: A company needs to borrow $150,000. For tax and related reasons, the company wants to pay 7.3% interest on this loan. There are three lenders for this money. The first charges 6%, the second charges 7% and the third charges 10%. The company is going to borrow twice as much from the first lender as from the third. How much should the company borrow from each lender?

\[
\begin{array}{cccc}
\text{in loan} & \text{int. rate} & \text{interest ($) } \\
1 & x & 0.06 & 0.06x \\
2 & y & 0.07 & 0.07y \\
3 & 0.10 & 0.10z \\
\hline
150,000 & 0.06x & 150,000 & 0.093 \\
\end{array}
\]

\[
\begin{align*}
\text{(A)} & \quad x + y + z = 150,000 \\
\text{(B)} & \quad 0.06x + 0.07y + 0.10z = 150,000(0.093) \\
\text{(C)} & \quad x - 2z = 0
\end{align*}
\]

Using substitution, \( x = 2z \), we get

\[
\begin{align*}
\text{(A)} & \quad y + 3z = 150,000 \\
\text{(B)} & \quad 7y + 22z = 1,095,000
\end{align*}
\]

\[
\begin{pmatrix}
-7 & -21 & -10500000 \\
1 & 3 & 1500000 \\
7 & 22 & 1,095,000
\end{pmatrix}
= \begin{pmatrix}
1 & 3 & 150,000 \\
0 & 1 & 45,000 \\
0 & -3 & -135,000
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & 15,000 \\
0 & 1 & 45,000
\end{pmatrix}
\]

\[
\begin{align*}
y &= 15,000 \\
z &= 45,000 \\
x &= 90,000
\end{align*}
\]