Math 1090 ~ Business Algebra

Section 2.4 Inverse Matrices

Objectives:
• Use Gauss-Jordan techniques to find an inverse of a matrix, if it exists.
• Use inverse matrices to solve systems of equations.

\[ 5x - 2y \leq 75 \]

\[
\begin{pmatrix}
  a & b \\
  c & d
\end{pmatrix}
\]

\[ S = Pe^t \]

\[ APY = (1 + \frac{r}{n})^n - 1 \]
Inverse Matrix

A⁻¹, read "A inverse," is a matrix such that
- A⁻¹ · A = I = A · A⁻¹
- A⁻¹ can only exist for a square matrix

(I = identity matrix that's same size as A)

Ex 1: Find A⁻¹ for

a) A = \[
\begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}
\]

Method to find A⁻¹ for A
1. If A is not square, A⁻¹ does not exist (DNE).
2. If A is square,
   a) Augment A with the identity matrix.
   b) Perform elementary row operations on the augmented matrix until the left side is I, the identity matrix.
   c) What is on the right side is A⁻¹.

b) A = \[
\begin{bmatrix}
5 & 4 \\
0 & 2
\end{bmatrix}
\]

\[A^{-1} = \frac{1}{5(2) - 4(0)} \begin{bmatrix}
2 & -4 \\
0 & 5
\end{bmatrix} = \frac{1}{10} \begin{bmatrix}
2 & -4 \\
0 & 5
\end{bmatrix}\]

Formula for A⁻¹ of a 2x2 matrix (if A⁻¹ exists)
If \[A = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix},\]
then \[A^{-1} = \frac{1}{ad-bc} \begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix}.

Check: \[AA^{-1} = \begin{bmatrix}
5 & 4 \\
0 & 2
\end{bmatrix} \begin{bmatrix}
\frac{1}{5} & -\frac{2}{5} \\
0 & \frac{1}{2}
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} = I\]
Ex 2: Find $A^{-1}$ if possible.

a) $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ -5 & 1 & 0 & 1 \\ 3 & -2 & 7 & 0 \end{bmatrix}$

$3 \times 4$ matrix which is NOT square

$\Rightarrow A^{-1}$ DNE

b) $A = \begin{bmatrix} 7 & -4 & 6 \\ -4 & 7 & 5 \\ 2 & -1 & 1 \end{bmatrix}$

$(3 \times 3$ matrix)

$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -2 & 4 \\ 3 & -5 & 7 \\ 1 & -1 & 0 \end{bmatrix}$

A Note: If, in this process, you get a 0 0 0 row on left side, it means $A^{-1}$ DNE.
Ex 3: Use $A^{-1}$ from Example 2(b) to solve this system of equations.

$$\begin{align*}
7x - 4y + 6z &= 1 \\
7x - 4y + 5z &= 0 \\
2x - y + z &= 7
\end{align*}$$

To solve $AX = B$

(\text{where } A \text{ is an } n \times n \text{ matrix})

$X$ is an $n \times 1$ column vector of variables

$B$ is an $n \times 1$ column vector of constants

we can left-multiply both sides by $A^{-1}$. $A^{-1}AX = A^{-1}B$

$IX = A^{-1}B.$

$$X = A^{-1}B$$

another example:

Solve $\begin{align*}
7x - 4y + 6z &= 5 \\
7x - 4y + 5z &= 1 \\
2x - y + z &= -1
\end{align*}$

$\Rightarrow B = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$

(note: $A$ is same as above $\Rightarrow A^{-1}$ is same also.)

$X = A^{-1}B$

$$= \begin{bmatrix} 1 & -2 & 4 \\ 3 & -5 & 7 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 - 2 - 4 \\ 15 - 5 - 7 \\ 5 - 1 + 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$$