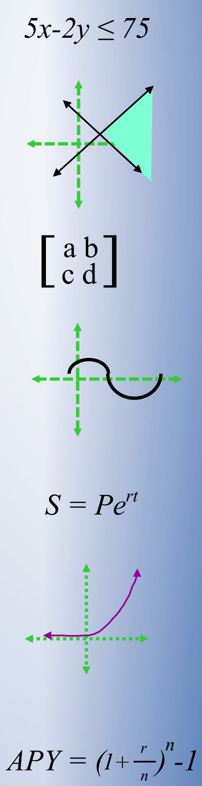

Math 1090 ~ Business Algebra

Section 2.2 Matrix Multiplication

Objectives:

- Determine whether two matrices can be multiplied together.
- Multiply two matrices.
- Write an Identity matrix of the proper size.

Definitions

Matrix Multiplication AB

Given A (size $m \times n$) and B (size $n \times p$) AB is an $m \times p$ matrix with ij entry given by $a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$. i.e. the product/sum of the i^{th} row of A with the j^{th} column of B.

(# cols of A must = # rows of B, to do AB)

Identity Matrix I

I is always a square matrix which has 1 in each diagonal entry and zeros everywhere else.

2×2 $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 3×3 $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$AI = A$

Properties of Matrix Multiplication

1. (AB)C = A(BC) associativity
2. A(B + C) = AB + AC
3. (B + C)A = BA + CA

distributivity

4. $(AB)^T = B^T A^T$

$\boxed{AB \neq BA}$

matrix multiplication is NOT commutative; order of multiplication matters

Ex 1: Given $A = \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \\ -1 & 0 \end{bmatrix}$

2×3 $\quad$ 3×2

Find AB and BA, if possible.

① $AB = \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1(1)+0(3)+4(-1) & 1(2)+0(-2)+4(0) \\ 5(1)+1(3)+2(-1) & 5(2)+1(-2)+2(0) \end{bmatrix}$

$2\times 3 \quad 3 \times 2$

cols A = # rows B

$\Rightarrow$ we can find AB (a 2×2 matrix)

$= \begin{bmatrix} -3 & 2 \\ 6 & 8 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 2 \\ 3 & -2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1(1)+2(5) & 1(0)+2(1) & 1(4)+2(2) \\ 3(1)+-2(5) & 3(0)+-2(1) & 3(4)+-2(2) \\ -1(1)+0(5) & -1(0)+0(1) & -1(4)+0(2) \end{bmatrix}$

$3\times 2 \quad 2 \times 3$

cols B = # rows of A

$\Rightarrow$ BA possible; it will be 3×3 matrix

$= \begin{bmatrix} 11 & 2 & 8 \\ -7 & -2 & 8 \\ -1 & 0 & -4 \end{bmatrix}$

Ex 2: Is $(AA^T)^T = A^TA$? no

we know $(AB)^T = B^T A^T$

$$\Rightarrow (\underbrace{A}_{①}\underbrace{A^T}_{②})^T = \underbrace{(A^T)^T}_{②^T} \cdot \underbrace{A^T}_{①^T} = AA^T \neq A^TA$$

Ex 3: Given $A = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 0 & 1 \\ 0 & 0 & -4 \end{bmatrix}$, find A^2.

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 0 & 1 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 3 \\ 2 & 0 & 1 \\ 0 & 0 & -4 \end{bmatrix}$$

$3 \times 3 \quad 3 \times 3$

$$= \begin{bmatrix} 0(0)+1(2)+3(0) & 0(1)+1(0)+3(0) & 0(3)+1(1)+3(-4) \\ 2(0)+0(2)+1(0) & 2(1)+0(0)+1(0) & 2(3)+0(1)+1(-4) \\ 0(0)+0(2)+-4(0) & 0(1)+0(0)+-4(0) & 0(3)+0(1)+-4(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -11 \\ 0 & 2 & 2 \\ 0 & 0 & 16 \end{bmatrix}$$

Ex 4: Solve for x.

$$\begin{bmatrix} -5 \\ 10 \\ -19 \end{bmatrix} + \begin{bmatrix} -2x \\ 13 \\ -8 \end{bmatrix} + \begin{bmatrix} -8 \\ -4 \\ -7 \end{bmatrix} = \begin{bmatrix} 5 \\ 19 \\ -34 \end{bmatrix}$$

(a matrix equation)

$$\begin{bmatrix} -5-2x-8 \\ 10+13+-4 \\ -19-8-7 \end{bmatrix} = \begin{bmatrix} 5 \\ 19 \\ -34 \end{bmatrix}$$

$$\begin{bmatrix} -13-2x \\ 19 \\ -34 \end{bmatrix} = \begin{bmatrix} 5 \\ 19 \\ -34 \end{bmatrix}$$

$$\Rightarrow -13-2x = 5$$

$$-2x = 18$$

$$x = -9$$

Ex 5: Solve for x and y.

$$\begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 5 \end{bmatrix}$$

$(2\times 2)\ (2\times 1)\quad 2\times 1$

$$\begin{bmatrix} 1x + 3y \\ -1x + 5y \end{bmatrix} = \begin{bmatrix} 11 \\ 5 \end{bmatrix}$$

Two eqns:

① $x + 3y = 11$

② $-x + 5y = 5$

$+$

$8y = 16$

$y = 2$

① $x + 3(2) = 11$

$x + 6 = 11$

$x = 5$

solution:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$