Math 1090 ~ Business Algebra

Section 1.1 Linear Equations in One Variable

Objectives:
- Simplify linear expressions.
- Solve linear equations.
- Express a rational equation as a linear equation.
- Translate a word problem into a linear equation.
Vocabulary

equation (like a complete sentence)

two expressions set equal to each other
ex: \(5x + 3 = 2x - 1\)
we solve eqns.

expression (like a fragment of a sentence)

collection of algebraic terms
ex: \(3x^2 + 5x + 6\)
we simplify expressions

identity

an equation (eqn) that’s true for all values of the variable (i.e. we do not solve an identity because it has \(\infty\) solutions)

equivalent equations

if we start w/ a balance scale, we keep scale balanced
i.e. eqns are like balanced scales & we do “same thing” to both sides of eqn to keep it “balanced” or equivalent
Solve equations and simplify expressions

Ex 1: Solve

a) \(3x + 22 = 7x + 2\)

\[-3x \quad -3x\]

\[20 = 4x + 2\]

\[\frac{20}{4} = \frac{4x}{4} \quad \Rightarrow \quad 5 = x\]

b) \(\frac{2}{3}x - 1 = \frac{x - 2}{2}\)

\[6\left(\frac{2}{3}x - 1\right) = \left(\frac{x - 2}{2}\right)^3\]

\[4x - 6 = 3x - 6 \quad \Rightarrow \quad 4x - 3x = 6 - 6\]

\[x = 0\]

Ex 2: Simplify

a) \(3(x-1) + 2x + 5 - 7\)

\[= 3x - 3 + 2x + 2\]

\[= 5x - 5\]

b) \(4 - (2x + 5) + 6 + 5(x - 3)\)

\[= 4 - 2x - 5 + 6 + 5x - 15\]

\[= 3x - 10\]
Vocabulary

rational equation: an eqn that has the variable in the denominator of a fraction (where denominator & numerator are polynomials)

domain
set of all variable values that are allowed

Ex 3: Solve these rational equations by turning them into linear equations.

Note: Check the domain.

a) \( \frac{2x}{x-3} = 4 + \frac{6}{x-3} \)

\[
(x-3)(\frac{2x}{x-3}) = (4 + \frac{6}{x-3})(x-3)
\]

\( 2x = 4(x-3) + \frac{6(x-3)}{x-3} \)

\( 2x = 4x - 12 + 6 \)

\( 2x = 4x - 6 \)

\( -2x = -6 \)

\( x = 3 \), not allowed in domain

\[ \boxed{N.S.} \]

b) \( \frac{3 - 1}{x} = 4 - \frac{2 + 1}{3x} \)

\[
\frac{3 + 1}{x} = 4 - \frac{2 + 1}{3x}
\]

\( 36x + 3 \cdot 12x = 2(12x) + 12x \)

\( 36 + 3x = 8x + 12 \)

\( 36 = 5x + 12 \)

\( 24 = 5x \)

\( \frac{24}{5} = x \)
Ex 4: Suppose a professor counts the final exam as being equal to each of the other tests in her course, and she will also change the lowest test score to match the final exam if that is higher. If a student's four test scores are 83, 67, 52 and 90, what is the lowest score the student can earn on the final exam and still obtain an 80 average for the course?

\[ x = \text{final exam score} \]

\[
\frac{83 + 67 + x + 90 + x}{5} = 80
\]

\[
\frac{240 + 2x}{5} = (80)5
\]

\[
240 + 2x = 400
\]

\[
x = 80
\]
Ex 5: Three less than 4 times a number is 25. What is the number?

\[ x = \text{the number} \]

\[ 4x - 3 = 25 \]

\[ 4x = 28 \]

\[ x = 7 \]

Ex 6: The perimeter of a rectangle is 700 ft and the length of the rectangle is four times as long as the width. Find the dimensions of the rectangle.

\[ P = 700 \text{ ft} \]

\[ 4x + x + 4x + x = 700 \]

\[ 10x = 700 \]

\[ x = 70 \]

dimensions: 70 ft by 280 ft