Math 1060 ~ Trigonometry

9 Applications of Radian Measure

Learning Objectives

In this section you will:

- Determine arc length.
- Determine area of a sector of a circle.
- Solve problems involving linear and angular velocity.

\[
\sin^2 u + \cos^2 u = 1 \\
\sin 2u = 2 \sin u \cos u \\
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \\
c^2 = a^2 + b^2 - 2ab \cos C
\]
Vocabulary

Arc: (length of) part of a circle
Sector: like a "pie piece"; portion of a circle bounded by 2 radii and intercepted arc
Length of a circular arc

$$s = r\theta$$  (note: $$\theta$$ must be in radians)

Ex 1: Find the arc length along a circle of radius 10 cm subtended by an angle of 125°.

$$s = r\theta$$

$$s = 10\ cm \left(\frac{25\pi}{18}\right)$$

$$s = \frac{125\pi}{18}\ cm \approx 21.8\ cm$$

Ex 2: What is the radius of a circle for which 2/3 of the circumference is 6π ft?

$$s = 6\pi\ ft$$

$$s = r\theta$$

$$6\pi = r \left(\frac{4\pi}{3}\right)$$

$$\frac{3}{4}\left(\frac{4\pi}{3}\right) = r$$

$$4.5\ or\ \frac{9}{2} = r$$

$$r = 4.5\ ft$$
Area of a Sector

Area of whole circle = \( \pi r^2 \)

\[ \Rightarrow \text{area of sector} = \left( \frac{\theta}{2\pi} \right) \pi r^2 = \frac{1}{2} \theta r^2 \]

\[ \text{area of sector of circle w/ angle } \theta \]

\[ \text{Note: } \theta \text{ must be in radians!} \]

Ex 3: A lawn sprinkler sprays a distance of 15 feet out and rotates back and forth at a 120° angle. What is the area that the sprinkler waters?

\[ A = \frac{1}{2} \Theta r^2 \]

\[ A = \frac{1}{2} \left( \frac{2\pi}{3} \right) (15 \text{ ft})^2 \]

\[ A = \frac{\pi}{3} (225) \text{ ft}^2 \]

\[ A = 75\pi \text{ ft}^2 \]

\[ \approx 235.6 \text{ ft}^2 \]
Linear and Angular Velocity

- Velocity can be positive or negative (to indicate direction)
  - Angular velocity: CC = clockwise
  - " "      : counterclockwise

Average Velocity: \( \overline{v} = \frac{\text{distance}}{\text{time}} = \frac{s}{t} = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right) \)

Read "\( \overline{v} \) bar"

Average Angular Velocity: \( \overline{\omega} = \frac{\text{change in angle}}{\text{time}} = \frac{\theta}{t} \)

Read "\( \omega \) bar"

Speed = |\( \overline{v} \)|

Magnitude or absolute value of velocity; has no information about direction

Instantaneous speed (for both linear and angular velocity) just means how fast it's going at one moment in time.
Velocity for Circular Motion

\[ v = r \omega \]

*from last page*

\[ v = r \left( \frac{\Theta}{t} \right) = rw \]

Ex 4: The giant wheel in London, known as the Millennium Wheel has a radius of 60 meters. It completes one rotation in 30 minutes. What is the linear and angular velocity of a person riding in one of the cabins on the wheel? (It does not stop to pick up passengers, they hop on and off as it moves.)

(since it goes the same speed all the time, the instantaneous velocity = avg. velocity)

\[ v = rw \]

1. What is \( \omega \)?

\[ \omega = \frac{\Theta}{t} = \frac{2\pi}{30 \text{ min}} = \frac{\pi}{15} \approx 0.209 \text{ rad/min} \]

2. What is \( v \)?

\[ v = 60 \text{ m} \left( \frac{\pi}{15 \text{ min}} \right) = 4\pi \text{ m/min} \approx 12.6 \text{ m/min} \]