Math 1060 ~ Trigonometry

7 Graphing The Cosine and Sine Functions

Learning Objectives

In this section you will:

- Graph the cosine and sine functions.
- Learn the properties of the cosine and sine functions, including domain and range, period, phase shift, amplitude and vertical shift.
- Identify cosine and sine functions as periodic functions.
- Determine whether a periodic function is even or odd.
- Use properties to graph periodic functions.
- Write an equation from the graph of a sine or cosine function.
\[ f(x) = \sin x \]

http://tube.geogebra.org/student/m45345?mobile=true
Graph of $f(x) = \sin x$

Domain: $(-\infty, \infty)$
Period: $2\pi$

Range: $[-1, 1]$ (repeats itself in shape)
Symmetry: wrt origin. (odd fn)
\[ f(x) = \cos x \]

http://tube.geogebra.org/student/m45354?mobile=true
Graph of $f(x) = \cos x$

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Period: $2\pi$

Symmetry: wrt y-axis (even fn)
How can you graph \( y = 2 \sin(x - \frac{\pi}{3}) + 1 \)?

This is a transformation of the basic \( y = \sin x \) curve.
It may help to remember transformations to one of the algebraic functions.

How does the graph of \( y = -3(x+2)^2 - 1 \) relate to the graph of \( y = x^2 \)?

In general, remember the effect of \( a, h \) and \( k \) on the graph of \( y = x^2 \).
\[
y = a(x-h)^2 + k
\]

| \((h,k)\) new vertex \(\begin{align*}
  h &= \text{horiz. shift} \\
  k &= \text{vert. shift}
\end{align*}\) |
| \(|a| = \text{vert. "stretch" factor} \begin{align*}
  &\text{if } |a| > 1, \text{ stretch} \\
  &\text{if } |a| < 1, \text{ shrink}
\end{align*}\) |
| \(\{ \begin{align*}
  &\text{if } a > 0, \text{ no vert. reflection (concave up)} \\
  &\text{if } a < 0, \text{ vert. reflection (concave down)}
\end{align*}\) |
\[ y = A \sin(b(x-h)) + k \]

What effect do \( A, b, h \) and \( k \) have on the graph of trigonometric functions?

Let's look at it one part at a time: \( y = A \sin x \)

- **Amplitude**: \( |A| \)
  
  \[ y = \sin x, \quad y = \cos x \]
  
  \[ A = 1 \]

  amplitude = max distance (vertically) traveled from the horizontal axis of oscillation; it's half the distance from highest \( y \)-value to lowest \( y \)-value.

Ex 1: Graph each of these.

\[ y = 3\sin x \quad A = 3 \]

\[ y = -2\cos x \quad A = 2 \]

- multiplying by \( A \)
  
  (on the outside of the fn) causes a vertical stretch/shrink

- \( A \) is the vertical stretch by factor \( A \)

  and vertical reflection
Periodic Functions

A function is periodic if there is a real number $p$ so that $f(x+p) = f(x)$. The smallest positive number $p$, if it exists is called the period of $f$.

\[ y = \sin(bx) \]

- Period = horiz. distance before graph repeats itself.

(\text{normally for } y = \sin x \text{ and } y = \cos x \text{ period } = 2\pi)

Ex 2: Graph each of these.

\[ y = \sin(2x) \]

\text{period } = \frac{2\pi}{2} = \pi

\[ y = \cos(\frac{1}{2} x) \]

\text{period } = \frac{2\pi}{\frac{1}{2}} = 4\pi

amplitude = 1 \quad \text{and } (0, 0)

amplitude = 1 \quad \text{and } (0, 1)
\[ y = \sin(x-h) \]

- Horizontal shift (phase shift) = \( h \)

Ex 3: Graph each of these.

\[ y = \sin(x+\pi) \]

\[ \text{horiz. shift} = -\pi \]

\[ \text{amp.} = 1 \]
\[ \text{period} = 2\pi \]

Note: This is same as \( y = -\sin x \)

\[ y = \cos(x - \frac{\pi}{2}) \]

\[ \text{horiz. shift} = \frac{\pi}{2} \]

\[ \text{amp} = 1 \]
\[ \text{period} = 2\pi \]

Note: This is same as \( y = \sin x \)

Note: \( x-h=0 \)
\[ x=h \]
\[ y = \sin(b(x - h)) \]

- Period = \( \frac{2\pi}{b} \)
- Horizontal shift = \( h \)

**WARNING:**

Must be in form \( y = \sin(b(x-h)) \) to decide horiz shift.

Ex 4: Graph each of these.

\[ y = \sin(2x - \pi) \]

\[ \begin{align*}
  y &= \sin(2(x - \frac{\pi}{2})) \\
  \text{Period} &= \frac{2\pi}{2} = \pi, \text{horiz. shift} = \frac{\pi}{2}
\end{align*} \]

\[ \begin{align*}
  \text{amp} &= 1
\end{align*} \]

\[ y = \cos\left(\frac{1}{2}x + \frac{\pi}{2}\right) \]

\[ \begin{align*}
  y &= \cos\left(\frac{1}{2}(x + \pi)\right) \\
  \text{Period} &= \frac{2\pi}{\frac{1}{2}} = 4\pi, \text{horiz. shift} = -\pi
\end{align*} \]

\[ \begin{align*}
  \text{amp} &= 1
\end{align*} \]
$y = \sin(x) + k$

Vertical Shift = $k$

Ex 5: Graph each of these.

$y = \sin x - 2$

Shift down 2

$y = \cos x + 1$

Shift up 1

amp = 1
period = $2\pi$
So, when we graph a sine or cosine function there are these things to consider:

Ex 6: List the transformations of this function.

\[ y = 3 \cos(2x - \pi) + 1 = 3 \cos \left(2 \left( x - \frac{\pi}{2} \right) \right) + 1 \]

- **Amplitude**
  \[ 3 \]
  \[
  \sqrt{\frac{2\pi}{2}} = \frac{\pi}{2}
  \]
- **Phase shift (horizontal)**
  \[ \frac{\pi}{2} \text{ (right)} \]
- **Vertical shift**
  \[ 1 \text{ (up)} \]

Ex 7: List the transformations of this function. \( f(x) = -2 \sin(4x - \pi) - 2. \)

- **Amplitude**
  \[ |\text{-}2| = 2 \]
- **Period**
  \[ \frac{2\pi}{4} = \frac{\pi}{2} \]
- **Phase shift (horizontal)**
  \[ \frac{\pi}{4} \text{ (right)} \]
- **Vertical shift**
  \[ -2 \text{ (down)} \]
- **Reflection: Vertical**
Ex 8: Analyze the transformations and write a function equation of this graph using the cosine function and then one using the sine function.

1. Period: \( \frac{2\pi}{2} = \pi \)
   Amplitude: 3
   Horizontal shift: 0
   Vertical shift: 1
   \( y = 3\cos(2x) + 1 \)

2. Period = \( \pi \)
   Amp = 3
   Vertical shift = 1
   Horizontal shift = \( -\frac{\pi}{4} \)

   \[ x = \frac{-\pi}{4} \implies x + \frac{\pi}{4} = 0 \]
Here are some applets in case you want to play with the transformation variables.

http://www.analyzemath.com/trigonometry/sine.htm

http://tube.geogebra.org/student/m45354?mobile=true

Here are instructions and the equation format from the text for graphing a periodic (sinusoidal) function.

For $\omega > 0$, the functions

$$C(x) = A\cos(\omega x + \phi) + B \quad \text{and} \quad S(x) = A\sin(\omega x + \phi) + B$$

- have period $\frac{2\pi}{\omega}$
- have amplitude $|A|
- have phase shift $-\frac{\phi}{\omega}$
- have vertical shift $B$