Math 1060 ~ Trigonometry

6 Beyond The Unit Circle

Learning Objectives

In this section you will:

- Determine the values of the six trigonometric functions from the coordinates of a point on a circle, centered at the origin, with any radius $r$.
- Solve related application problems.
Determining Sine and Cosine

Consider the acute angle $\theta$ drawn in standard position.

$Q(x,y)$ is a point on the terminal side of $\theta$ which lies on the circle $x^2 + y^2 = r^2$.

$P(x',y')$ is a point on the terminal side of $\theta$ which lies on the Unit Circle.

Theorem: If $Q(x,y)$ is a point on the terminal side of an angle $\theta$, plotted in standard position, which lies on the circle $x^2 + y^2 = r^2$, then

$x = r \cos \theta$ and $y = r \sin \theta$.

$$\cos \theta = \frac{x}{r} = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\sin \theta = \frac{y}{r} = \frac{y}{\sqrt{x^2 + y^2}}$$

From these it is possible to determine all of the other four functions.
Ex 1: Determine the sine, secant and tangent of an angle which contains the point $Q(3, -2)$ when plotted in standard position.

$$r = \sqrt{(3-0)^2+(2-0)^2} = \sqrt{3^2+2^2} = \sqrt{13}$$

$$\sin \theta = \frac{y}{r} = \frac{-2}{\sqrt{13}}$$

$$\cos \theta = \frac{x}{r} = \frac{3}{\sqrt{13}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\sqrt{13}}{3}$$

$$\tan \theta = \frac{y}{x} = \frac{-2}{3}$$

Ex 2: If the terminal side of $\theta$ lies on the line $3x - 4y = 0$ in the third quadrant, find the values of the six trigonometric functions of $\theta$ by finding a point on the line.

$$r = \sqrt{y^2+3^2} = \sqrt{25} = 5$$

$$\sin \theta = \frac{y}{r} = \frac{-3}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{-4}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{-3}{-4}$$

$$\csc \theta = \frac{1}{\sin \theta} = -\frac{5}{3}$$

$$\sec \theta = \frac{1}{\cos \theta} = -\frac{5}{4}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{-4}{3}$$
Ex 3: Determine the radius of the circle of revolution for Salt Lake City, which is located at a latitude of 40.76° N. Assume the radius at the equator to be 3960 miles.

\[ \theta = 40.76° \quad x = \text{radius of SLC circle of revolution we want to know } x = ? \]
\[ r = 3960 \text{ mi} \]

\[ \cos \theta = \frac{x}{r} \quad \Rightarrow \quad x = r \cos \theta \]

\[ x = 3960 \cos(40.76°) \]

\[ x \approx 2999.5 \text{ mi} \]

**WARNING:** Be sure your calculator is in degree mode!