The Unit Circle

Consider the Unit Circle, \( x^2 + y^2 = 1 \), with angle \( \theta \) in standard position and the corresponding arc measuring \( s \) units in length.

\[ s = r\theta \]

To identify real numbers with oriented angles, we “wrap” the real number line around the Unit Circle and associate to each real number \( t \) an oriented arc on the unit circle with initial point \((1,0)\).
Ex 1: Sketch the oriented arc on the Unit Circle corresponding to each of these real numbers.

a) $t = \frac{3\pi}{4}$

b) $t = -3\pi$

c) $t = 2$

d) $t = -\frac{\pi}{2}$

Determining the cosine and sine functions as points on the Unit Circle.

Ex 2:

a) Label the quadrant angles above in radians and degrees and determine the cosine and sine of each.

b) $\cos (-\pi) =$
Question: If the hypotenuse of an isosceles right triangle is 1 unit, how long are each of the legs?

Ex 3:

a) On the figure above, label all the points on the Unit Circle corresponding with angles which are multiples of $\frac{\pi}{4}$.

b) $\sin\frac{5\pi}{4} = \ldots$

Question: If the hypotenuse of a right 30°-60°-90° triangle is 1 unit, how long are each of the legs?

Ex 4:

a) On the figure above, label all the points on the Unit Circle corresponding with angles which are multiples of $\frac{\pi}{6}$.

b) $\cos\frac{5\pi}{6} = \ldots$

c) $\sin\frac{2\pi}{3} = \ldots$
A complete Unit Circle looks like this.

Given the symmetry of the Unit Circle and the Reference Angle Theorem, you can determine cosine and sine values of these common angles readily.

**Reference Angle**

A reference angle for a non-terminal angle, \( \theta \), is that angle made up of the terminal side of \( \theta \) and the \( x \)-axis.

- It is always positive.
- It is always acute.

**Reference Angle Theorem:** Suppose \( \alpha \) is the reference angle for \( \theta \). Then \( \cos \theta = \pm \cos \alpha \) and \( \sin \theta = \pm \sin \alpha \), where the sign is determined by the quadrant in which the terminal side of \( \theta \) lies.

Ex 5: For each of the following angles, determine the reference angle and the sine and cosine of each.

<table>
<thead>
<tr>
<th>Angle (degrees)</th>
<th>Reference Angle</th>
<th>( \cos(\theta) )</th>
<th>( \sin(\theta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0°</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>30°</td>
<td>( \frac{\pi}{6} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>45°</td>
<td>( \frac{\pi}{4} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
</tr>
<tr>
<td>60°</td>
<td>( \frac{\pi}{3} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
</tr>
<tr>
<td>90°</td>
<td>( \frac{\pi}{2} )</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Ex 5 a) \( \frac{2\pi}{3} \)

Ex 5 b) \( -\frac{5\pi}{6} \)

Ex 5 c) 270°

Ex 5 d) -315°
The Pythagorean Identity

For any angle, $\theta$, $\cos^2 \theta + \sin^2 \theta = 1$.

Ex 6: Using the given information about $\theta$, find the indicated value.

a) If $\theta$ is a second quadrant angle, such that $\sin \theta = \frac{3}{4}$, find $\cos \theta$.

b) If $\theta$ is between $\pi$ and $\frac{3\pi}{2}$ and $\cos \theta = \frac{1}{2}$, find $\sin \theta$.

c) If $\sin \theta = 1$, find $\cos \theta$. 