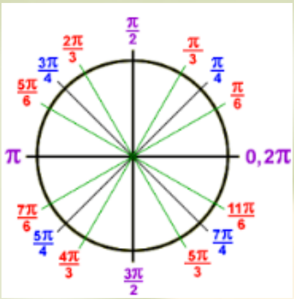
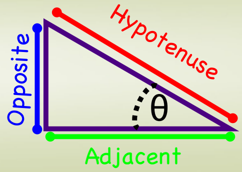
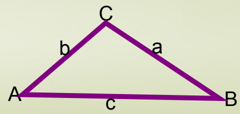
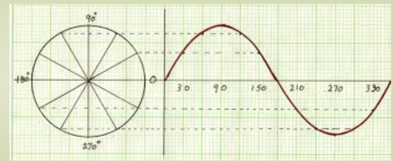








$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2\sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Math 1060 ~ Trigonometry

2 Right Triangles

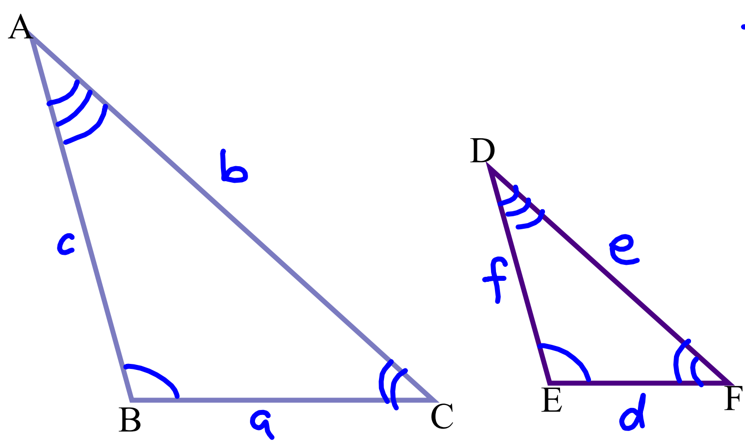
Learning Objectives

In this section you will:

- **Identify the trigonometric ratios.**
- **Learn the trigonometric ratio values for 30º, 45º and 60º.**
- **Solve right triangles and related application problems.**

Similar Triangles

Two triangles are similar if they have the same shape, more specifically, if their corresponding angles are congruent. Additionally two triangles are similar if and only if their corresponding sides are proportional.



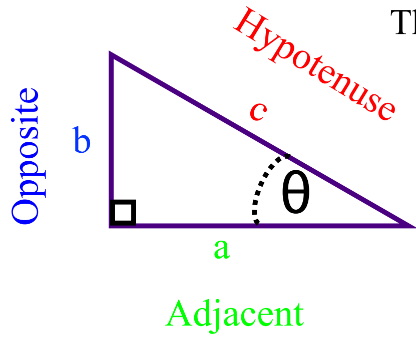
$$\frac{b}{c} = \frac{e}{f}$$

$$\frac{a}{d} = \frac{b}{e}$$

etc.

<u>Trigonometric Ratios</u>

Consider this generic right triangle with angle θ.



The six trigonometric ratios are:

- $\sin(\theta) = \dfrac{opposite}{hypotenuse} = \dfrac{b}{c}$
- $\cos(\theta) = \dfrac{adjacent}{hypotenuse} = \dfrac{a}{c}$
- $\tan(\theta) = \dfrac{opposite}{adjacent} = \dfrac{b}{a}$
- $\cot(\theta) = \dfrac{adjacent}{opposite} = \dfrac{a}{b}$
- $\sec(\theta) = \dfrac{hypotenuse}{adjacent} = \dfrac{c}{a}$
- $\csc(\theta) = \dfrac{hypotenuse}{opposite} = \dfrac{c}{b}$

$\underbrace{SOH}\underbrace{CAH}\underbrace{TOA}$

$\cot\theta = \dfrac{1}{\tan\theta}$, $\csc\theta = \dfrac{1}{\sin\theta}$,

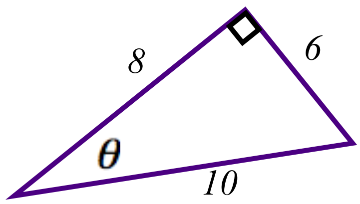
$\sec\theta = \dfrac{1}{\cos\theta}$

<u>Important properties of the trigonometric ratios</u>:

a. For all right triangles with the same acute angle θ, because they are similar, the values of the resulting trigonometric ratios of θ will be identical.

b. Cosecant, secant and cotangent are <u>reciprocal ratios</u> of sine, cosine and tangent respectively.

Ex 1: Find the six trigonometric ratios for the angle, θ.



$\sin\theta = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{8}{10} = \frac{4}{5}$

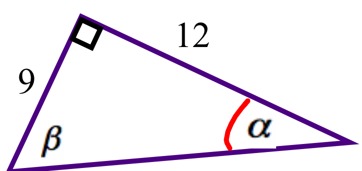
$\tan\theta = \frac{6}{8} = \frac{3}{4}$

$\cot\theta = \frac{4}{3}$

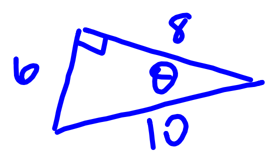
$\csc\theta = \frac{5}{3}$

$\sec\theta = \frac{5}{4}$

Ex 2: Verify that this triangle is similar to the one in example 1 and find the six trigonometric ratios for the angle which corresponds to θ.



from ex 1:



check for similarity: $\frac{9}{6} = \frac{3}{2}$ and $\frac{15}{10} = \frac{3}{2}$ ✓

$\frac{12}{8} = \frac{3}{2}$

$\sin\alpha = \frac{9}{15} = \frac{3}{5}$

$\cos\alpha = \frac{12}{15} = \frac{4}{5}$

$\tan\alpha = \frac{9}{12} = \frac{3}{4}$

$\csc\alpha = \frac{5}{3}$

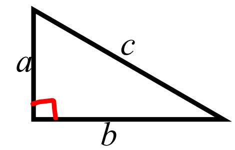
$\sec\alpha = \frac{5}{4}$

$\cot\alpha = \frac{4}{3}$

Pythagorean Theorem

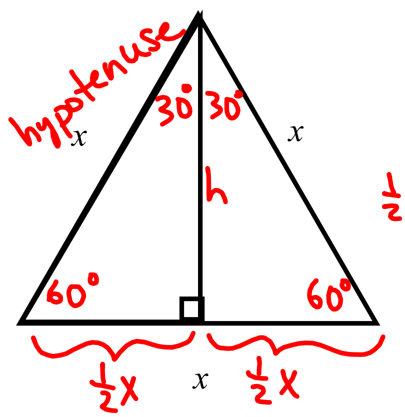
The square of the hypotenuse in a right triangle is equal to the sum of the squares of the two shorter sides. For example, in this right triangle, with hypotenuse c

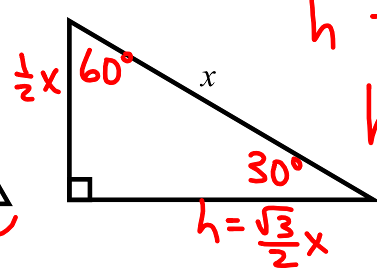
$\boxed{c^2 = a^2 + b^2.}$



Trigonometric Ratios of 30º, 60º, 90º Triangles

We begin with an equilateral triangle and cut it in half.





h = ? use Pythagorean Thm

$$h^2 + \left(\tfrac{1}{2}x\right)^2 = x^2$$

$$h^2 + \tfrac{1}{4}x^2 = x^2$$

$$h^2 = \tfrac{3}{4}x^2$$

$$h = \sqrt{\tfrac{3}{4}x^2}$$

$$h = \frac{\sqrt{3}\,x}{2}$$

$$\sin 30° = \frac{\tfrac{1}{2}\cancel{x}}{\cancel{x}} = \frac{1}{2} \qquad \sec 30° = \frac{1}{\cos 30°} = \frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}$$

$$\cos 30° = \frac{\tfrac{\sqrt{3}}{2}\cancel{x}}{\cancel{x}} = \frac{\sqrt{3}}{2} \qquad \csc 30° = \frac{1}{\sin 30°} = 2$$

$$\tan 30° = \frac{\tfrac{1}{\cancel{2}}\cancel{x}}{\tfrac{\sqrt{3}}{\cancel{2}}\cancel{x}} = \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{3} \qquad \cot 30° = \frac{1}{\tan 30°} = \sqrt{3}$$

Note: for this and future math classes, you should know these values.

You will also need to know:

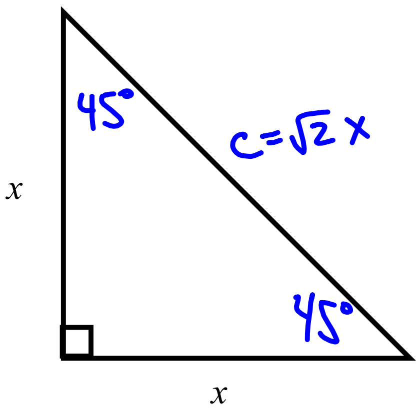
$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \sqrt{3}$

Trigonometric Ratios of 45º, 45º, 90º Triangles

use Pythagorean Thm



$$x^2+x^2=c^2$$
$$2x^2=c^2$$
$$\sqrt{2x^2}=c$$
$$\sqrt{2}x=c$$

Note: these trig. ratio values should be on your note card.

$$\sin 45^\circ = \frac{\cancel{x}}{\sqrt{2}\cancel{x}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{\cancel{x}}{\sqrt{2}\cancel{x}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{x}{x} = 1$$

$$\sec 45^\circ = \frac{1}{\cos 45^\circ} = \sqrt{2}$$

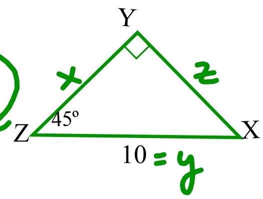
$$\csc 45^\circ = \frac{1}{\sin 45^\circ} = \sqrt{2}$$

$$\cot 45^\circ = \frac{1}{\tan 45^\circ} = 1$$

Ex 3: Find the missing parts of these right triangles.

a) $x = 5\sqrt{2}$

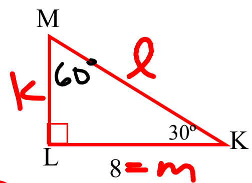
b) $z = x = 5\sqrt{2}$



$\cos 45^\circ = \frac{x}{10}$

$x = 10\cos 45^\circ = 10\left(\frac{\sqrt{2}}{2}\right) = 5\sqrt{2}$

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$



c) $k = \frac{8\sqrt{3}}{3}$

d) $l = \frac{16}{\sqrt{3}}$

$\cos 30^\circ = \frac{8}{l}$

$\tan 30^\circ = \frac{k}{8}$

$k = 8\tan 30^\circ = 8\left(\frac{\sqrt{3}}{3}\right) = \frac{8\sqrt{3}}{3}$

$l = \frac{8}{\cos 30^\circ} = \frac{8}{\sqrt{3}/2} = \frac{16}{\sqrt{3}} = \frac{16\sqrt{3}}{3}$

e) $t = 5$

$12^2 + t^2 = 13^2$

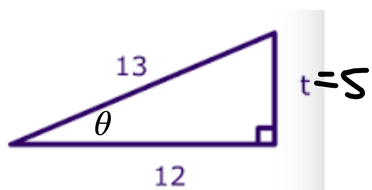
$144 + t^2 = 169$

$t^2 = 25$

$t = 5$

f) $\sin\theta = \frac{5}{13}$

$\sin\theta = \frac{5}{13}$



(this is Pythagorean triple 5-12-13 right triangle)

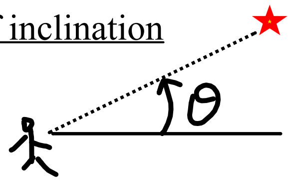
g) $\sec\theta = 13/12$

$\sec\theta = \frac{hyp}{adj} = \frac{13}{12}$

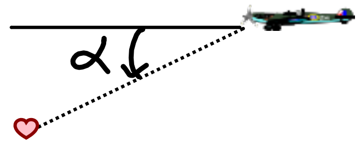
h) $\tan\theta = 5/12$

$\tan\theta = \frac{5}{12}$

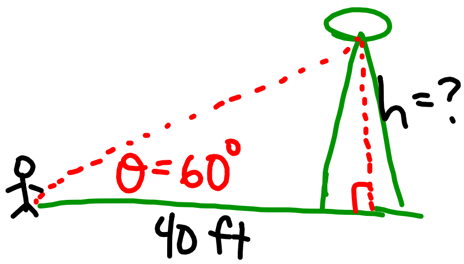
Angle of inclination



Angle of depression



Ex 4: The angle of inclination from a point on the ground 40 feet from the base of a tower is 60º. How tall is the tower?



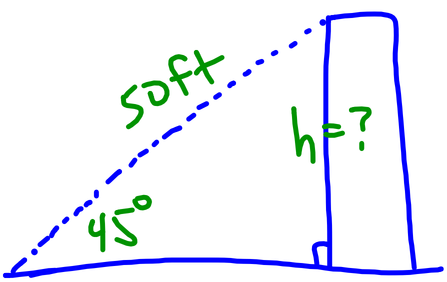
$$\tan 60^\circ = \frac{h}{40}$$

$$h = 40 \tan 60^\circ$$

$$= 40(\sqrt{3}) = 40\sqrt{3} \text{ ft}$$

$$\approx 69.3 \text{ ft}$$

Ex 5: If a 50-foot tight-rope from the corner of the top of a building meets the ground at an angle of 45º, how tall is the building?



$$\sin 45^\circ = \frac{h}{50}$$

$$h = 50 \sin 45^\circ$$

$$= 50\left(\frac{\sqrt{2}}{2}\right) = 25\sqrt{2} \text{ ft}$$

$$\approx 35.4 \text{ ft}$$