

Math 1060 ~ Trigonometry

$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2 \sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

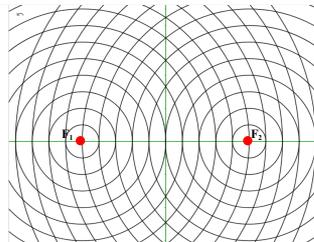
28 Conic Sections: Hyperbolas

Learning Objectives

In this section you will:

- Define a hyperbola in a plane.
- Determine whether an equation represents a hyperbola or some other conic section.
- Graph a hyperbola from a given equation.
- Determine the center, vertices, foci and eccentricity of a hyperbola.
- Find the equation of a hyperbola from a graph or from stated properties.

Ex 1: Given the points $F_1(-5,0)$ and $F_2(5,0)$, plot several points such that the difference of the distances from F_1 and F_2 to each point is 4. Draw the curve connecting the points.



Hyperbolas

General form: $Ax^2 + By^2 + Cx + Dy + E = 0$, where A and B have opposite signs.

Given: two points (foci) and a distance (c).

Definition: A hyperbola is the set of all points in a plane such that for each point on the hyperbola, the difference of its distances from two fixed points is constant.

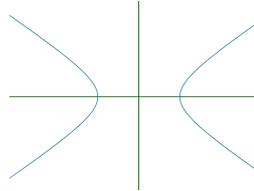
Vocabulary

Transverse axis

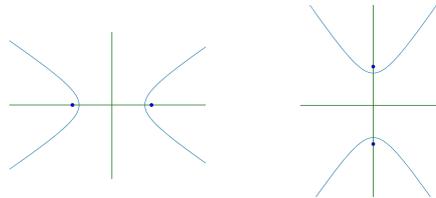
Asymptotes

Center

Foci

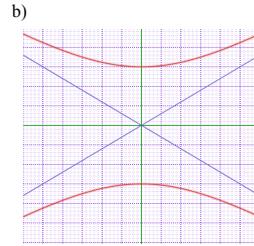
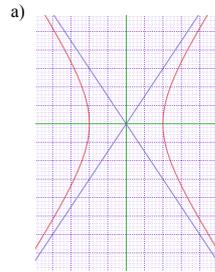


Standard Form of an Equation of a Hyperbola with Center at (0,0)



The variables a , b and c have a special relationship.

Ex 2: Write the equation of these hyperbolas in standard form.



Ex 3: Determine the value of c for each hyperbola above and plot the foci.

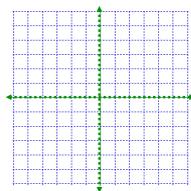
Translations of a Hyperbola

Standard Hyperbola
center at $(0,0)$

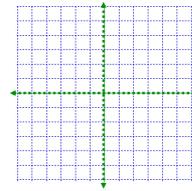
Translated Hyperbola
center at (h,k)

Ex 4: Sketch each of these curves and locate the foci.

a) $4x^2 - 9y^2 = 36$

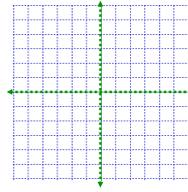


b) $16(y+2)^2 - 9(x-3)^2 = 36$

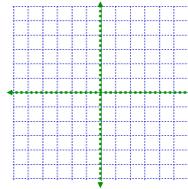


Ex 5: Write an equation and sketch each of these.

a) The hyperbola such that the center is $(-2,3)$, one of the asymptotes passes through $(1,4)$ and it is vertically oriented.

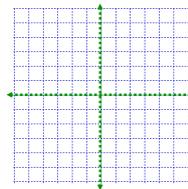


b) A hyperbola with vertices at $(-4,3)$ and $(2,3)$ and foci at $(-6,3)$ and $(4,3)$



Ex 6: Write this equation in standard form, sketch it, including the foci.

$$x^2 - 9y^2 - 4x - 18y - 14 = 0$$



Eccentricity of a Hyperbola

$$e = c/a$$

Ex 7: Identify each of these equations as one of these:

C - Circle

E - Ellipse that is not a circle (longer in which direction)

H - Hyperbola (facing which way)

P - Parabola (facing which way)

i) $9x^2 - 4y^2 - 36x + 8y - 4 = 0$

ii) $y^2 + 4x - 2y - 11 = 0$

iii) $16x^2 + 16y^2 + 64x - 32y - 176 = 0$

iv) $-9x^2 + 25y^2 - 54x - 50y - 281 = 0$

v) $9x^2 + 4y^2 - 18x + 16y - 11 = 0$

vi) $x^2 - 6x + 8y - 7 = 0$

vii) $2x^2 + 3y^2 + 12x + 24y + 60 = 0$