

**Math 1060 ~ Trigonometry**

**27 Conic Sections: Ellipses, Including Circles**

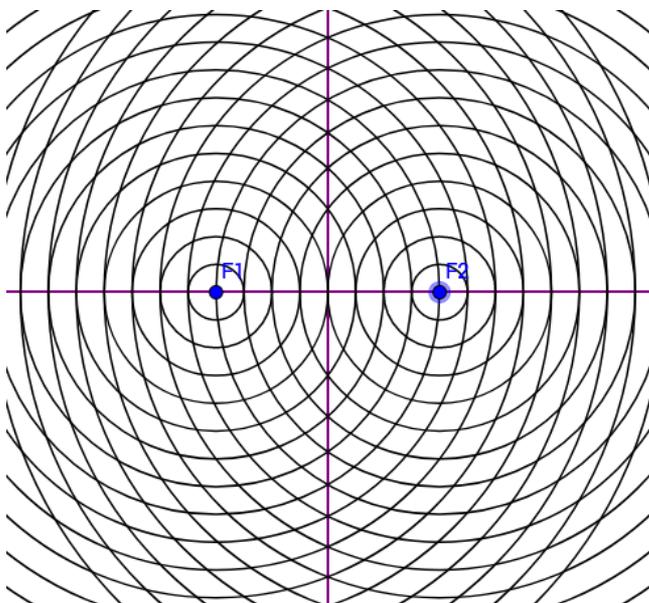
**Learning Objectives**

In this section you will:

- Define an ellipse in a plane.
- Determine whether an equation represents an ellipse.
- Graph an ellipse from a given equation.
- Determine the center, vertices, foci and eccentricity of an ellipse.
- Find the equation of an ellipse from a graph or from stated properties.

$\sin^2 u + \cos^2 u = 1$   
 $\sin 2u = 2 \sin u \cos u$   
 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$   
 $c^2 = a^2 + b^2 - 2ab \cos C$

Ex 1: Given the points  $F_1(-4,0)$  and  $F_2(4,0)$ , plot several points such that the sum of the distances from  $F_1$  and  $F_2$  to each point is 12. Draw the curve connecting the points.



## Ellipses

General form:  $Ax^2 + By^2 + Cx + Dy + E = 0$

(A and B have  
same sign)

Given: two points (foci) and a distance ( $c$ ).

Definition: An ellipse is the set of all points in a plane such that for each point on the ellipse, the sum of its distances from two fixed points is constant.

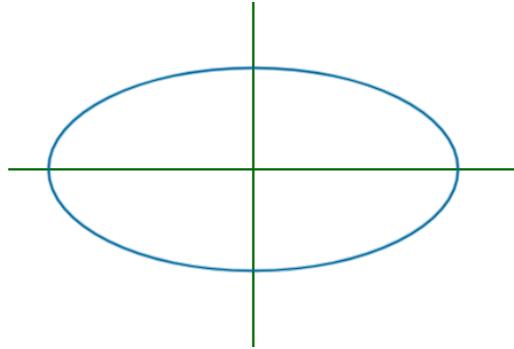
### Vocabulary

Major axis

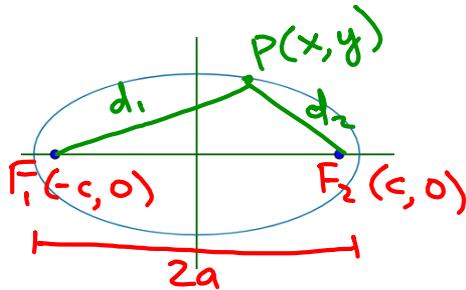
Minor axis

Center

Foci



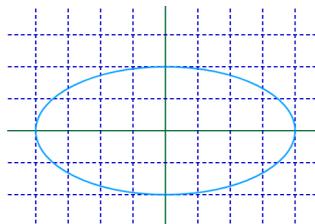
### Standard Form of an Equation of an Ellipse with Center at (0,0)



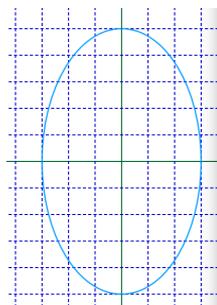
$$d_1 + d_2 = 2a$$

Ex 2: Write the equation of these ellipses in standard form.

a)



b)



The variables  $a$ ,  $b$  and  $c$  have a special relationship.

Ex 3: Determine the value of  $c$  for each ellipse above and plot the foci.

### Translations of an Ellipse

Standard Ellipse

center at  $(0,0)$

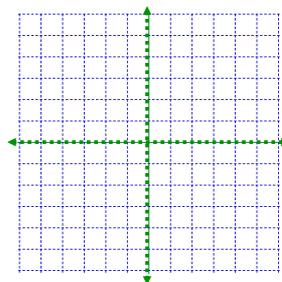
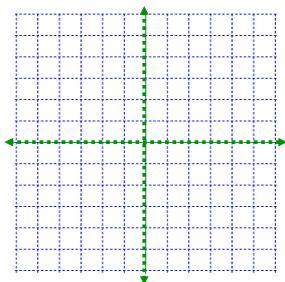
Translated Ellipse

center at  $(h,k)$

Ex 4: Sketch each of these curves and locate the foci.

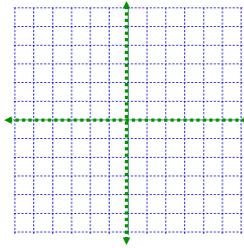
a)  $36x^2 + 16y^2 = 576$

b)  $9(x+2)^2 + 16(y-3)^2 = 144$

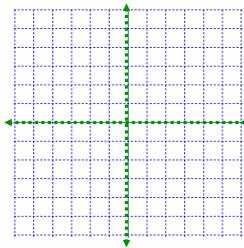


Ex 5: Write an equation and sketch each of these.

a) An ellipse with center point  $(-2,3)$ ,  $a = 5$ ,  $c = 3$ , longer in the vertical direction.

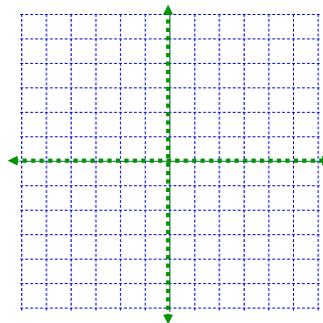


b) An ellipse with vertices at  $(-6,3)$  and  $(4,3)$  and foci at  $(-4,3)$  and  $(2,3)$



Ex 6: Write this equation in standard form, sketch it, including the foci.

$$x^2 + 9y^2 - 4x - 18y - 14 = 0$$



Eccentricity of an Ellipse

$$e = c/a$$