

Math 1060 ~ Trigonometry

27 Conic Sections: Ellipses, Including Circles

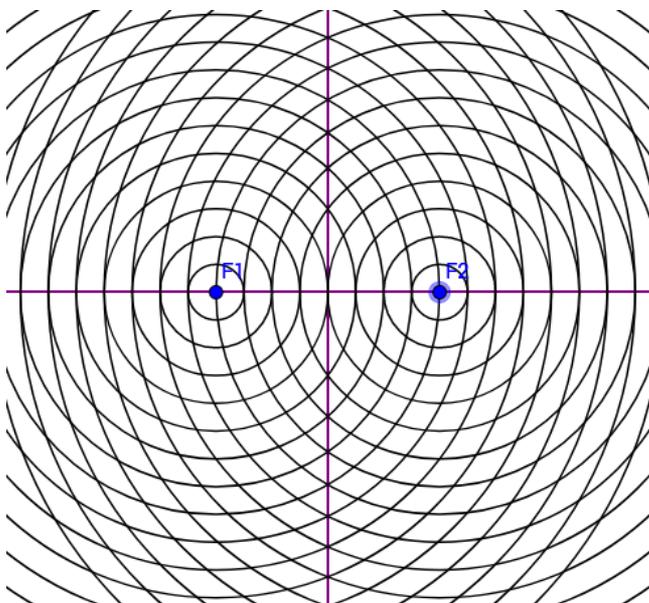
Learning Objectives

In this section you will:

- Define an ellipse in a plane.
- Determine whether an equation represents an ellipse.
- Graph an ellipse from a given equation.
- Determine the center, vertices, foci and eccentricity of an ellipse.
- Find the equation of an ellipse from a graph or from stated properties.

$\sin^2 u + \cos^2 u = 1$
 $\sin 2u = 2 \sin u \cos u$
 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
 $c^2 = a^2 + b^2 - 2ab \cos C$

Ex 1: Given the points $F_1(-4,0)$ and $F_2(4,0)$, plot several points such that the sum of the distances from F_1 and F_2 to each point is 12. Draw the curve connecting the points.



Ellipses

General form: $Ax^2 + By^2 + Cx + Dy + E = 0$

(A and B have
same sign)

Given: two points (foci) and a distance (c).

Definition: An ellipse is the set of all points in a plane such that for each point on the ellipse, the sum of its distances from two fixed points is constant.

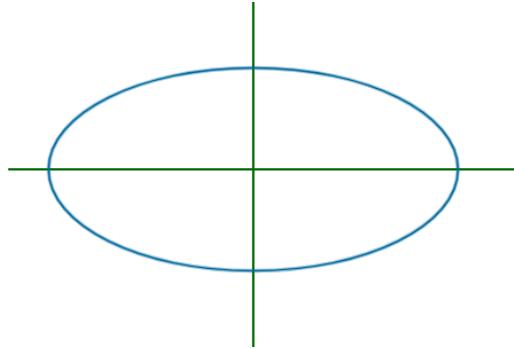
Vocabulary

Major axis

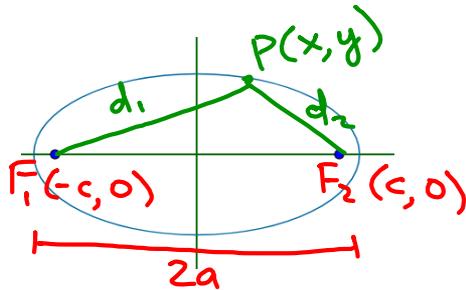
Minor axis

Center

Foci



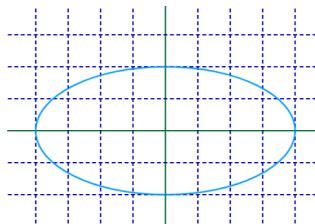
Standard Form of an Equation of an Ellipse with Center at (0,0)



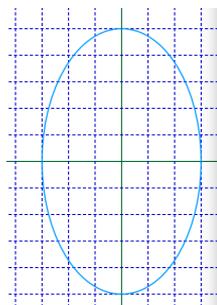
$$d_1 + d_2 = 2a$$

Ex 2: Write the equation of these ellipses in standard form.

a)



b)



The variables a , b and c have a special relationship.

Ex 3: Determine the value of c for each ellipse above and plot the foci.

Translations of an Ellipse

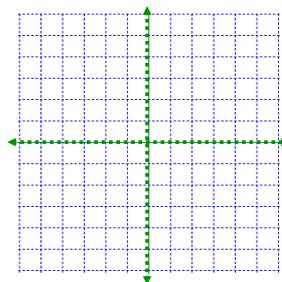
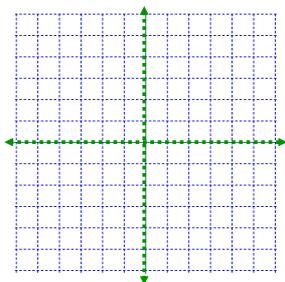
Standard Ellipse
center at $(0,0)$

Translated Ellipse
center at (h,k)

Ex 4: Sketch each of these curves and locate the foci.

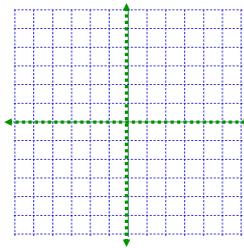
a) $36x^2 + 16y^2 = 576$

b) $9(x+2)^2 + 16(y-3)^2 = 144$

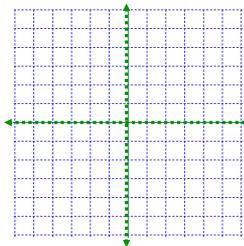


Ex 5: Write an equation and sketch each of these.

a) An ellipse with center point $(-2,3)$, $a = 5$, $c = 3$, longer in the vertical direction.

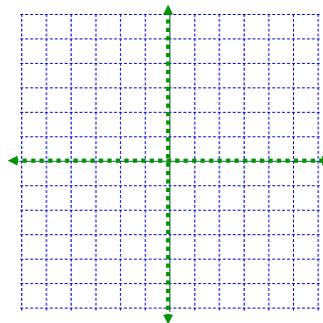


b) An ellipse with vertices at $(-6,3)$ and $(4,3)$ and foci at $(-4,3)$ and $(2,3)$



Ex 6: Write this equation in standard form, sketch it, including the foci.

$$x^2 + 9y^2 - 4x - 18y - 14 = 0$$



Eccentricity of an Ellipse

$$e = c/a$$