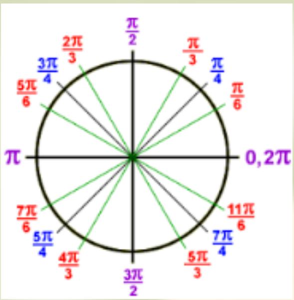
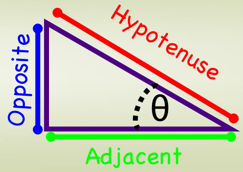
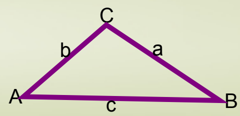
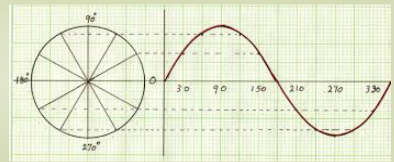








Math 1060 ~ Trigonometry

$\boxed{\sin^2 u + \cos^2 u = 1}$

$\boxed{\sin 2u = 2 \sin u \cos u}$

$\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}}$

$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$

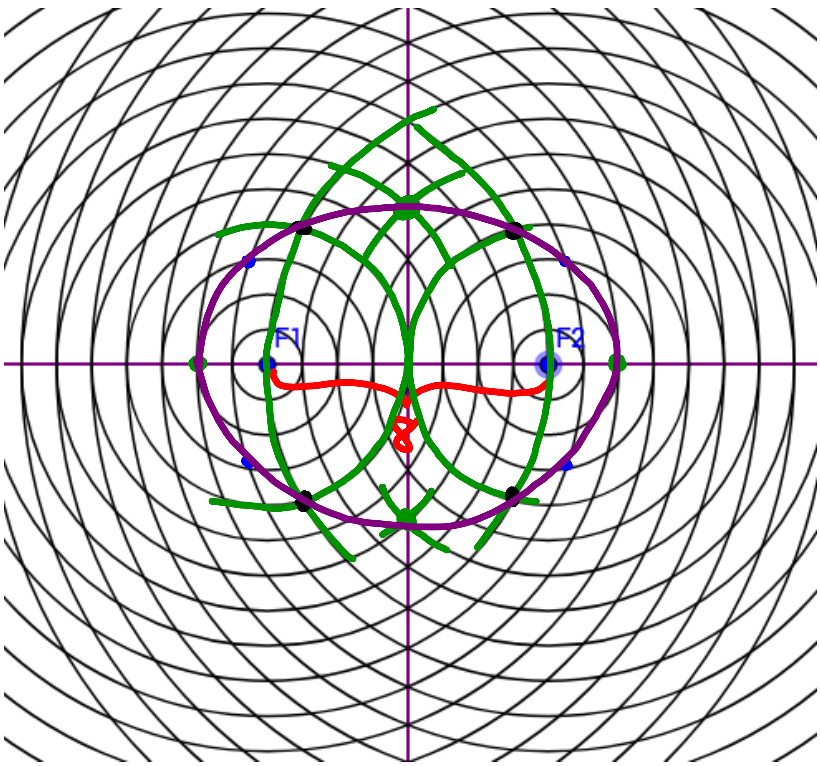
27 Conic Sections: Ellipses, Including Circles

Learning Objectives

In this section you will:

- Define an ellipse in a plane.
- Determine whether an equation represents an ellipse.
- Graph an ellipse from a given equation.
- Determine the center, vertices, foci and eccentricity of an ellipse.
- Find the equation of an ellipse from a graph or from stated properties.

Ex 1: Given the points $F_1(-4,0)$ and $F_2(4,0)$, plot several points such that the sum of the distances from F_1 and F_2 to each point is 12. Draw the curve connecting the points.



Ellipses A, B, C, D, E constants, $A \neq 0$ and $B \neq 0$

General form: $\boxed{Ax^2 + By^2 + Cx + Dy + E = 0}$ (A and B have same sign)

Given: two points (foci) and a distance (c).

Definition: An ellipse is the set of all points in a plane such that for each point on the ellipse, the sum of its distances from two fixed points is constant.

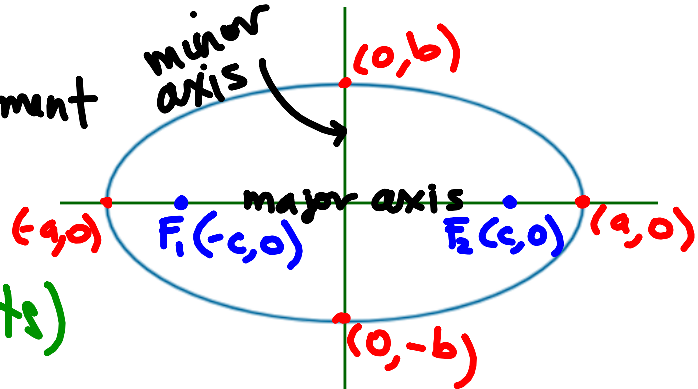
Vocabulary

Major axis: longer segment

Minor axis: shorter "

Center: intersectn of axes

Foci (the two fixed pts)



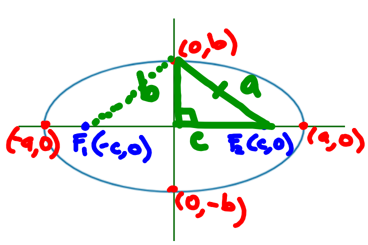
Note: ellipse has symmetry across both major and minor axes.

length of major axis $= 2a$

length of minor axis $= 2b$

distance between two foci $= 2c$

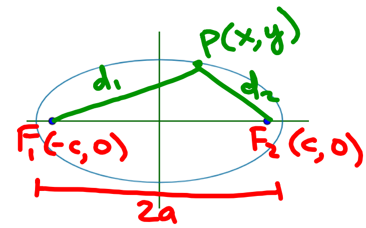
Note: If $a = b$, it's a circle.



$b^2 + c^2 = a^2$

$\boxed{b^2 = a^2 - c^2}$

Standard Form of an Equation of an Ellipse with Center at (0,0)



$$d_1 + d_2 = 2a$$

$$d_1 = \sqrt{(x-(-c))^2+(y-0)^2}$$

$$= \sqrt{(x+c)^2+y^2}$$

$$d_2 = \sqrt{(x-c)^2+y^2}$$

$$\Rightarrow \sqrt{(x+c)^2+y^2} + \sqrt{(x-c)^2+y^2} = 2a$$

$$\sqrt{x^2+2xc+c^2+y^2} + \sqrt{x^2-2xc+c^2+y^2} = 2a$$

$$\left(\sqrt{x^2+2xc+c^2+y^2}\right)^2 = \left(2a - \sqrt{x^2-2xc+c^2+y^2}\right)^2$$

$$\cancel{x^2}+2xc+\cancel{c^2}+\cancel{y^2} = 4a^2 - 4a\sqrt{x^2-2xc+c^2+y^2} + \cancel{x^2} - 2xc + \cancel{c^2} + \cancel{y^2}$$

$$2xc = 4a^2 - 4a\sqrt{x^2-2xc+c^2+y^2} - 2xc$$

$$-4a^2+2xc \quad -4a^2 \quad +2xc$$

$$(4xc-4a^2)^2 = \left(-4a\sqrt{x^2-2xc+c^2+y^2}\right)^2$$

$$16x^2c^2 - 2(16xca^2) + 16a^4 = 16a^2(x^2-2xc+c^2+y^2)$$

$$16x^2c^2 - 3\cancel{2xca^2} + 16a^4 = 16a^2x^2 - 3\cancel{2xca^2} + 16a^2c^2 + 16a^2y^2$$

$$x^2c^2 + a^4 = a^2x^2 + a^2c^2 + a^2y^2$$

$$x^2c^2 - a^2x^2 - a^2y^2 = a^2c^2 - a^4$$

$$\frac{x^2(c^2-a^2) - a^2y^2}{a^2(c^2-a^2)} = \frac{a^2(c^2-a^2)}{a^2(c^2-a^2)}$$

$$\frac{x^2}{a^2} - \frac{y^2}{c^2-a^2} = 1$$

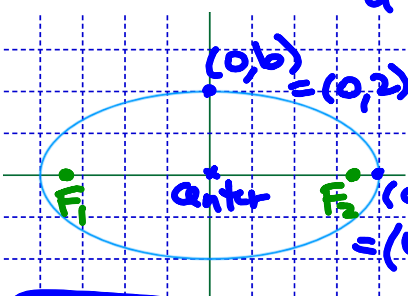
but remember: $b^2 = a^2 - c^2$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

ellipse formula w/ center at (0,0) (and major axis along x-axis)

Ex 2: Write the equation of these ellipses in standard form.

a) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

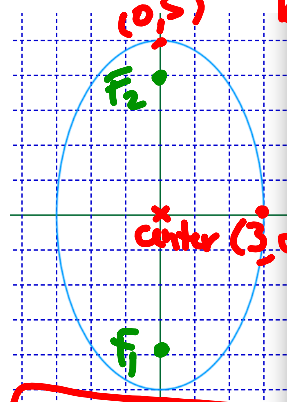


$(0,b) = (0,2) \Rightarrow b = 2$

$(a,0) = (4,0) \Rightarrow a = 4$

$\boxed{\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1}$

b) $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$



$a = 5$

$b = 3$

$\boxed{\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1}$

Ex 3: Determine the value of *c* for each ellipse above and plot the foci.

we know $b^2 = a^2 - c^2$

(a) $a = 4,\ b = 2$

$4 = 16 - c^2$

$c^2 = 12$

$c = 2\sqrt{3} \simeq 3.5$

(b) $a = 5,\ b = 3$

$9 = 25 - c^2$

$c^2 = 16$

$c = 4$

Translations of an Ellipse

Standard Ellipse

center at (0,0)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$\boxed{\text{major axis along horiz. axis}}$

Translated Ellipse

center at (h,k)

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Ex 4: Sketch each of these curves and locate the foci.

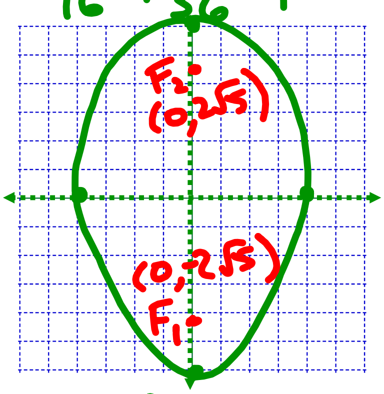
a) $36x^2 + 16y^2 = 576$

$$\frac{x^2}{16}+\frac{y^2}{36}=1$$

major axis along y-axis

$a=6$

$b=4$



$b^2=a^2-c^2 \Leftrightarrow c^2=a^2-b^2$

$c^2=36-16=20$

$c=2\sqrt{5}\approx 4.5$

b) $9(x+2)^2 + 16(y-3)^2 = 144$

$h=-2,\ k=3$

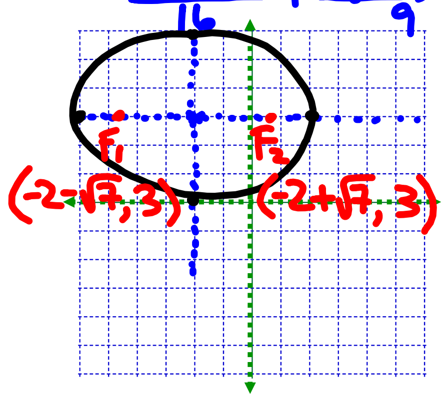
$$\frac{(x+2)^2}{16}+\frac{(y-3)^2}{9}=1$$

major axis is horizontal

center $(-2,3)$

$a=4$

$b=3$



$c^2=a^2-b^2$

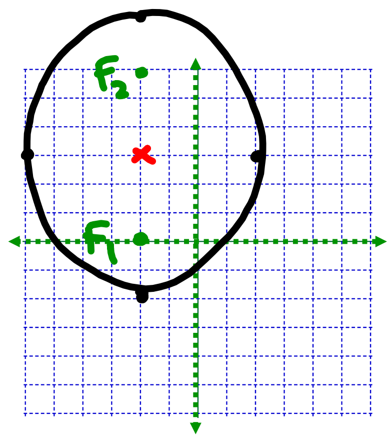
$c^2=4^2-3^2=7$

$c=\sqrt{7}\approx 2.6$

Ex 5: Write an equation and sketch each of these.

a) An ellipse with center point (-2,3), a = 5, c = 3, longer in the vertical direction.

$$b^2 = a^2 - c^2$$
$$= 25 - 9$$
$$= 16$$
$$\Rightarrow b = 4$$



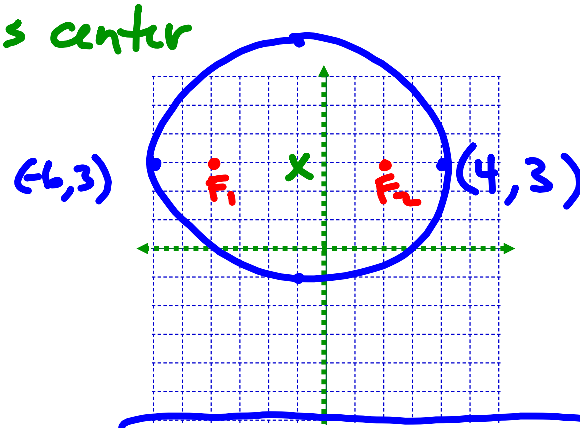
$$\boxed{\frac{(x+2)^2}{16} + \frac{(y-3)^2}{25} = 1}$$

b) An ellipse with vertices at (-6,3) and (4,3) and foci at (-4,3) and (2,3)

$2a = 10$

$a = 5$

(-1,3) is center



$c = 3$

$b^2 = 5^2 - 3^2 = 16$

$b = 4$

$$\boxed{\frac{(x+1)^2}{25} + \frac{(y-3)^2}{16} = 1}$$

Ex 6: Write this equation in standard form, sketch it, including the foci.

$x^2 + 9y^2 - 4x - 18y - 14 = 0$

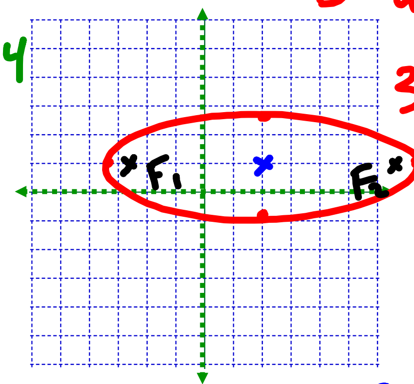
$e = \frac{2\sqrt{6}}{3\sqrt{3}} \approx 0.94$

$(x^2 - 4x \quad) + (9y^2 - 18y \quad) = 14$

$(x^2 - 4x + 4) + 9(y^2 - 2y + 1) = 14 + 4 + 9$

$(x-2)^2 + 9(y-1)^2 = 27$

$\Longleftrightarrow \quad \frac{(x-2)^2}{27} + \frac{(y-1)^2}{3} = 1$



$b^2 = a^2 - c^2$

$3 = 27 - c^2$

$c^2 = 24$

$c = \sqrt{24}$

$c = 2\sqrt{6}$

center: $(2, 1)$

$a = \sqrt{27} = 3\sqrt{3}$

$b = \sqrt{3}$

Eccentricity of an Ellipse

$\boxed{e = c/a}$

because $c < a$ for ellipses, $e < 1$ for ellipses

If $a = b$, we have a circle, in which case $c = 0 \Rightarrow e = 0$