Math 1060 ~ Trigonometry

26.5 Circles

Learning Objectives

In this section you will:

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> Define a circle in a plane.
> Determine whether an equation represents a circle.
> Graph a circle from a given equation.
> Determine the center and radius of a circle.
> Find the equation of a circle from stated properties.
A **circle** with center \((h,k)\) and radius \(r > 0\) is the set of all points \((x,y)\) in the plane whose distance to \((h,k)\) is \(r\).

**a Circle is the set of pts that are equidistant from a fixed pt., called the center.**

\[
\text{distance}^2 = (x-h)^2 + (y-k)^2
\]

\[
(r^2 = (x-h)^2 + (y-k)^2)
\]

The **Standard Equation of a Circle** with center at \((h,k)\) and radius \(r > 0\) is \((x-h)^2 + (y-k)^2 = r^2\).

**Ex 1:** Write an equation of a circle with center at \((2,-1)\) and radius 5.

\[
(x-2)^2 + (y+1)^2 = 5^2
\]

\[
(x-2)^2 + (y+1)^2 = 25
\]
Ex 2: Find the center and radius of the circle given by the equation

\[(x+4)^2 + (y+3)^2 = 9\]. Graph the circle.

\[(x-(-4))^2 + (y-(-3))^2 = 3^2\]

\[
\begin{array}{c}
\text{h} = -4 \\
\text{k} = -3 \\
\text{r} = 3
\end{array}
\]

Ex 3: Put this equation in standard form and graph the circle.

\[3x^2 + 3y^2 + 6x - 12y - 60 = 0\]

\[3(x^2+y^2+2x-4y-20)=0\]

\[(x^2+2x+1) + (y^2-4y+4) - 20 = 0\]

\[\left(\frac{x+1}{2}\right)^2 = 1 \quad \left(-\frac{y-2}{2}\right)^2 = 4\]

\((x^2+2x+1)+(y^2-4y+4) = 25\]

\[
\frac{(x+1)^2 + (y-2)^2}{5^2} \text{ standard form of circle eqn.}
\]

center: \((-1, 2)\)

radius: \(r = 5\)
Ex 4: Select the equations which might be a circle, put the equation in standard form and determine the center and radius.

a) \( x^2 + y^2 + 3x - 2y - 6 = 0 \)  
  b) \( x^2 + 6x - 2y + 6 = 0 \)  
  c) \( 2x^2 + 2y^2 - 4x - 10 = 0 \)

- \( (x-h)^2 + (y-k)^2 = r^2 \)

\( \uparrow \)

Cannot be a circle, because its missing \( y^2 \) term.

- \( x^2 + y^2 - 2x - 5 = 0 \)

Center: \((1,0)\)  \( r = \sqrt{6} \)

Ex 5: Write an equation of a circle with the points (-2,6) and (3,-1) as endpoints of the diameter.

1. Center:
   - midpoint between \((-2,6)\) and \((3,-1)\)
   - \( \left( \frac{-2+3}{2}, \frac{6+(-1)}{2} \right) = \left( \frac{1}{2}, \frac{5}{2} \right) \)

   \[ \text{circle:} \quad (x-\frac{1}{2})^2 + (y-\frac{5}{2})^2 = \frac{74}{4} \]

2. Radius:
   - \( r = \frac{1}{2} d \)
   - \( \frac{1}{2} \sqrt{(3-(-2))^2 + (-1-6)^2} = \frac{1}{2} \sqrt{5^2 + 7^2} = \frac{1}{2} \sqrt{74} \)
   - \( r^2 = \left( \frac{1}{2} \sqrt{74} \right)^2 \approx 4.3 \)