

Math 1060 ~ Trigonometry

22 The Unit Vector and Vector Applications

Learning Objectives

In this section you will:

- Use vectors in component form to solve applications.
- Find the unit vector in a given direction.
- Perform operations on vectors in terms of i and j .
- Use vectors to model forces.

$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2 \sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab \cos C$

The Unit Vector

It is often useful in solving problems to find a vector in the same direction as the given vector, but of magnitude 1. This is called a unit vector. If $\|\mathbf{v}\| = 1$, then $\hat{\mathbf{v}}$ is a unit vector.

If \mathbf{v} is a nonzero vector, then $\frac{\mathbf{v}}{\|\mathbf{v}\|}$ is a unit vector in the direction of \mathbf{v} .

Ex 1: Find a unit vector in the direction of $\mathbf{v} = \langle 3, -4 \rangle$.

The Principal Unit Vectors

It is also useful to have names for the principal unit vectors.

- The vector $\hat{\mathbf{i}}$ is defined by $\hat{\mathbf{i}} = \langle 1, 0 \rangle$.
- The vector $\hat{\mathbf{j}}$ is defined by $\hat{\mathbf{j}} = \langle 0, 1 \rangle$

This gives us another way to describe vectors. Thus $\langle a, b \rangle$ may also be $a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$.

Ex 2: Describe the vector from example 1 in terms of unit vectors.

Applications of Vectors

Vectors are useful in airplane navigation.

Ex 3: A plane is flying N 30° E at 400 mph and the wind is blowing west at 40 mph. What is the effective direction and speed of the plane?

- a) Draw a picture.
- b) Place the vectors for proper addition.
- c) Remember: the resultant is from the tail of the first to the tip of the last.
- d) Now do the math.

Vectors may be used to analyze forces acting on an object.

Ex 4: Two forces are pushing on an object, one exerts 30 lbs of pressure and a second exerts 20 lbs of pressure. The angle between the two forces is 70° . What is the resultant force on the object?

Ex 5: A 500-lb rock is being wheeled up a 30 degree ramp. What force is necessary to keep it from rolling back down the ramp? What is the weight the ramp is actually supporting?