Math 1060 ~ Trigonometry

22 The Unit Vector and Vector Applications

Learning Objectives

In this section you will:

- Use vectors in component form to solve applications.
- Find the unit vector in a given direction.
- Perform operations on vectors in terms of i and j.
- Use vectors to model forces.

\[
\begin{align*}
\sin^2 \theta + \cos^2 \theta &= 1 \\
\sin 2\theta &= 2 \sin \theta \cos \theta \\
\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
c^2 &= a^2 + b^2 - 2ab \cos \theta
\end{align*}
\]
The Unit Vector

It is often useful in solving problems to find a vector in the same direction as the given vector, but of magnitude 1. This is called a unit vector. If \( \|v\| = 1 \), then \( \hat{v} \) is a unit vector.

If \( v \) is a nonzero vector, then \( \frac{v}{\|v\|} \) is a unit vector in the direction of \( v \).

\[
\hat{v} = \frac{v}{\|v\|}
\]

Ex 1: Find a unit vector in the direction of \( v = \langle 3, -4 \rangle \).

\[
\hat{v} = \frac{\langle 3, -4 \rangle}{\|\langle 3, -4 \rangle\|} = \frac{\langle 3, -4 \rangle}{\sqrt{9 + 16}} = \frac{\langle 3, -4 \rangle}{\sqrt{25}} = \langle \frac{3}{5}, -\frac{4}{5} \rangle
\]

The Principal Unit Vectors

It is also useful to have names for the principal unit vectors.

• The vector \( \hat{i} \) is defined by \( \hat{i} = \langle 1, 0 \rangle \).
• The vector \( \hat{j} \) is defined by \( \hat{j} = \langle 0, 1 \rangle \).

This gives us another way to describe vectors. Thus \( \langle a, b \rangle \) may also be \( a \hat{i} + b \hat{j} \).

Ex 2: Describe the vector from example 1 in terms of unit vectors.

\[
\hat{v} = \langle 3, -4 \rangle = 3 \hat{i} - 4 \hat{j}
\]

\[
\hat{v} = \langle \frac{3}{5}, -\frac{4}{5} \rangle = \frac{3}{5} \hat{i} - \frac{4}{5} \hat{j}
\]
Applications of Vectors

Vectors are useful in airplane navigation.

Ex 3: A plane is flying N 30° E at 400 mph and the wind is blowing west at 40 mph. What is the effective direction and speed of the plane?

a) Draw a picture.

b) Place the vectors for proper addition.

c) Remember: the resultant is from the tail of the first to the tip of the last.

d) Now do the math.

we want $\|\vec{r}\|$ and the direction angle

we have SAS case $\Rightarrow$ use law of cosines

$\vec{r}^2 = 400^2 + 40^2 - 2(400)(40)\cos 60^\circ$

$\Rightarrow 145,600$

$\|\vec{r}\| = \sqrt{145,600} \approx 381.6 \text{ mph}$

use law of sines to find $\alpha$:

$\frac{\sin \alpha}{40} = \frac{\sin 60^\circ}{381.6}$

$\sin \alpha = 40 \frac{\sqrt{3}}{2} \frac{381.6}{381.6}$

$\alpha \approx 5.2^\circ$

$\Theta + d = 30^\circ \Rightarrow \theta \approx 30^\circ - 5.2^\circ = 24.8^\circ$

$\Rightarrow$ plane flies at $\approx 381.6 \text{ mph N} 24.8^\circ \text{E}$
Vectors may be used to analyze forces acting on an object.

Ex 4: Two forces are pushing on an object, one exerts 30 lbs of pressure and a second exerts 20 lbs of pressure. The angle between the two forces is 70°. What is the resultant force on the object?

\[ F = F_1 + F_2 \]

\[ F = \text{resultant force vector acting on the object} \]

\[ F = \langle 20 + 30 \cos 70°, -30 \sin 70° \rangle \]

\[ \approx \langle 30.26, -28.19 \rangle \]

\[ \|F\| \text{ is measured in lbs.} \]

\[ \|F\| \approx \sqrt{30.26^2 + (-28.19)^2} \approx 41.36 \text{ lbs} \]
Ex 5: A 500-lb rock is being wheeled up a 30 degree ramp. What force is necessary to keep it from rolling back down the ramp? What is the weight the ramp is actually supporting?

\[ \mathbf{u} + \mathbf{v} = \text{the 500-lb gravity/weight vector} \]

\[ \| \mathbf{u} \| = \text{wt that the ramp supports} \]

\[ \| \mathbf{v} \| = \text{wt pushing against rock along ramp (to keep it from rolling down)} \]

\[ \cos \theta = \frac{\| \mathbf{u} \|}{500} \]

\[ \| \mathbf{u} \| = 500 \cos 30^\circ = 500 (\frac{\sqrt{3}}{2}) = 250 \sqrt{3} \approx 433.01 \text{ lbs} \]

\[ \| \mathbf{v} \| = 500 \sin 30^\circ = 500 (\frac{1}{2}) = 250 \text{ lbs} \]