What do these equations represent?

\[ \theta = \beta \]

\[ r \cos \theta = a \]

\[ r \sin \theta = b \]
What about these?

\[ r = 2a \cos \theta \]

\[ r = 2b \sin \theta \]

\[ r = 2a \cos \theta + 2b \sin \theta \]

**Ex 1:**

\[
\begin{array}{c|cccccccc}
\emptyset & 0 & \pi/4 & \pi/3 & \pi/2 & 3\pi/4 & \pi & 5\pi/4 & 3\pi/2 & 7\pi/4 & 2\pi \\
\hline
r & & & & & & & & & & \\
\end{array}
\]

---

**Symmetry:**

- **Symmetry with respect to the line \( \theta = \pi/2 \):**
  - Replace \((r, \theta)\) with \((r, \pi - \theta)\) or \((-r, -\theta)\):
  - If an equivalent equation results, the graph has this type of symmetry.

- **Symmetry with respect to the polar axis \((\emptyset = 0)\):**
  - Replace \((r, \theta)\) with \((-r, \theta)\) or \((r, \pi - \theta)\):
  - If an equivalent equation results, the graph has this type of symmetry.

- **Symmetry with respect to the pole**
  - Replace \((r, \theta)\) with \((-r, \theta)\) or \((r, \pi + \theta)\):
  - If an equivalent equation results, the graph has this type of symmetry.

If a polar equation passes a symmetry test, then its graph definitely exhibits that symmetry. However, if a polar equation fails a symmetry test, then its graph may or may not have that kind of symmetry.
Zeros and maximum $r$-values

Other helpful tools in graphing polar equations are knowing the values for $\theta$ for which $r$ is maximum and those for which $r = 0$.

Ex 2: Graph $r = \frac{1}{2} \cos \theta$

Symmetry:

$|r|$ maximum:

Zero of $r$:

\[ r = a \pm b \cos \theta \quad a > 0, \ b > 0 \]
\[ r = a \pm b \sin \theta \]

\begin{array}{c}
\text{Limaçon} \\
\begin{array}{c}
\frac{a}{b} \geq 2 \\
\text{Convex limaçon} \\
\frac{a}{b} < 2 \\
\text{Dimpled limaçon} \\
\frac{a}{b} = 1 \\
\text{Cardioid} \\
\text{-always passes through pole} \\
\frac{a}{b} < 1 \\
\text{Limaçon with inner loop}
\end{array}
\end{array}\]
Ex 3: Graph \[ r = 3 \sin 2\theta \]

Symmetry:

\( r \) | maximum:

Zero of \( r \):

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>0</th>
<th>( \pi/8 )</th>
<th>( \pi/6 )</th>
<th>( \pi/4 )</th>
<th>( \pi/3 )</th>
<th>( \pi/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Roses

\[ r = a \sin(n \theta) \],

or

\[ r = a \cos(n \theta) \].

If \( n \) is odd, the rose is \( n \)-petalled. If \( n \) is even, the rose is \( 2n \)-petalled.

For \( n \) integer:

No reason to limit ourselves to \( n \) integer:

Or even rational:

\( n = \frac{\sqrt{2}}{2} \)