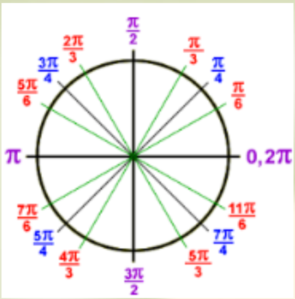
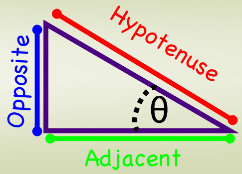
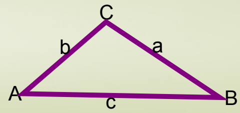
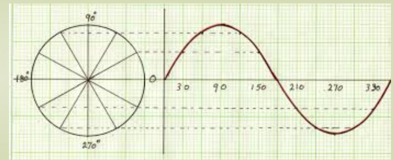








$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2 \sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Math 1060 ~ Trigonometry

18 Graphing Polar Equations

Learning Objectives

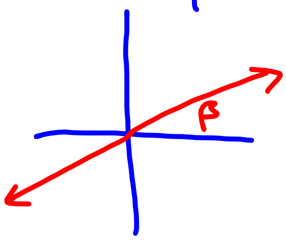
In this section you will:

- **Learn techniques for graphing polar equations.**
- **Graph polar equations.**

What do these equations represent?

These are all lines!

1. $\theta = \beta$ (β constant)

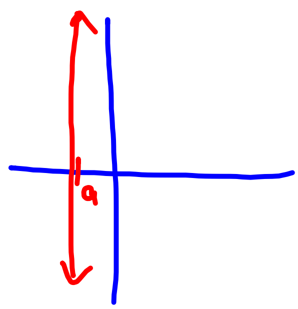


(radial line through the pole/origin)

2. $r\cos\theta = a$ $\iff$ $x = a$

(a constant)

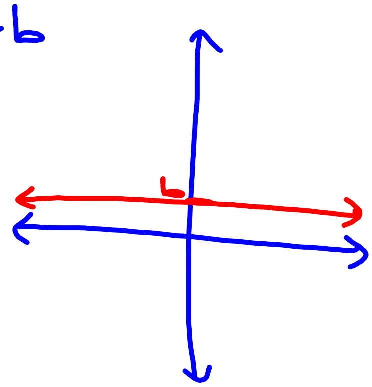
vertical line



3. $r\sin\theta = b$ $\iff$ $y = b$

(b constant)

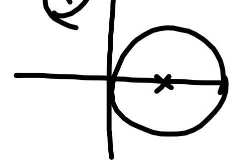
horizontal line



What about these? all of these eqns represent circles (that go thru origin)

① $r = 2a\cos\theta$ (a constant)

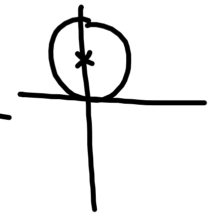
$r^2 = 2a(r\cos\theta) \iff x^2+y^2 = 2ax$



② $r = 2b\sin\theta$ (b constant)

circle centered at (0,b) w/ radius b

$x^2-2ax+y^2=0$ ②



$(x^2-2ax+a^2)+y^2=a^2$

③ $r = 2a\cos\theta + 2b\sin\theta$ (a,b constants)

circle centered at (a,b) w/ radius of $\sqrt{a^2+b^2}$

$(x-a)^2+y^2=a^2$

① circle centered at (a,0) w/ radius of a

Ex 1:

$r = 4\cos\theta$

θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
r	4	$2\sqrt{2}$	2	0	-2	$-2\sqrt{2}$	-4	$-2\sqrt{2}$	0	$2\sqrt{2}$	4

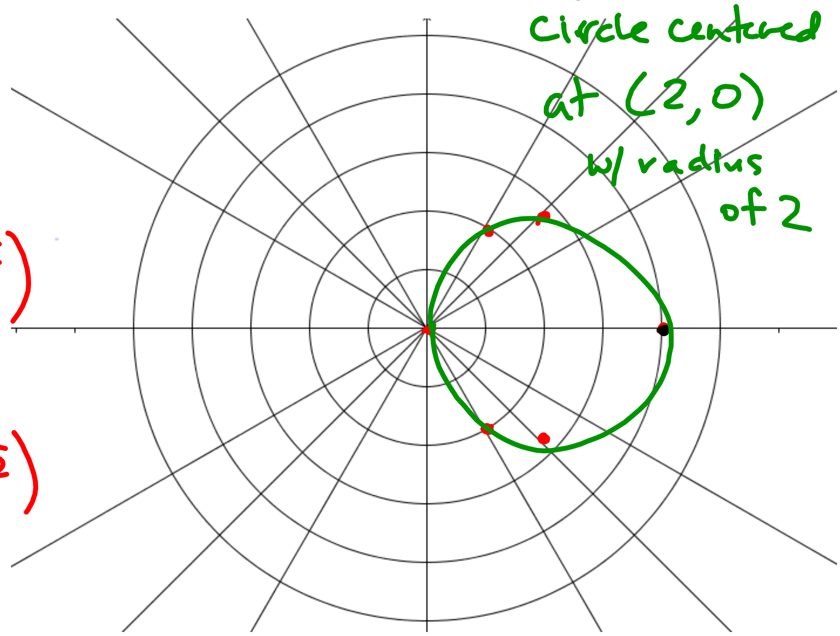
$4\left(\frac{\sqrt{2}}{2}\right)$

$(4,0) = (-4,\pi)$

$\left(2\sqrt{2},\frac{\pi}{4}\right) = \left(-2\sqrt{2},\frac{5\pi}{4}\right)$

$\left(0,\frac{\pi}{2}\right) = \left(0,\frac{3\pi}{2}\right)$

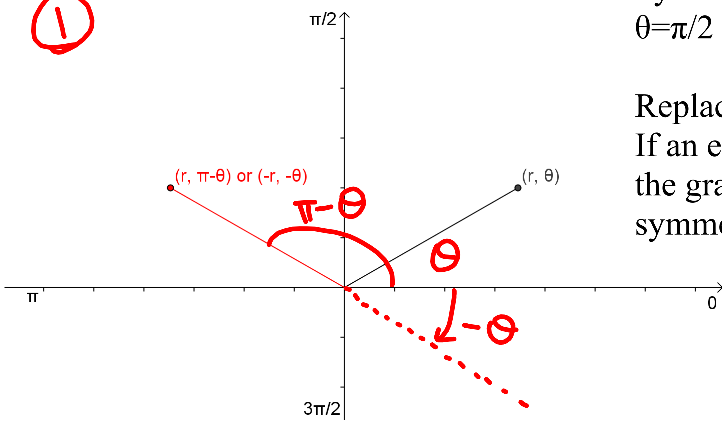
$\left(\frac{7\pi}{4},2\sqrt{2}\right) = \left(\frac{3\pi}{4},-2\sqrt{2}\right)$



circle centered at (2,0) w/ radius of 2

Symmetry

1



Symmetry with respect to the line $\theta=\pi/2$

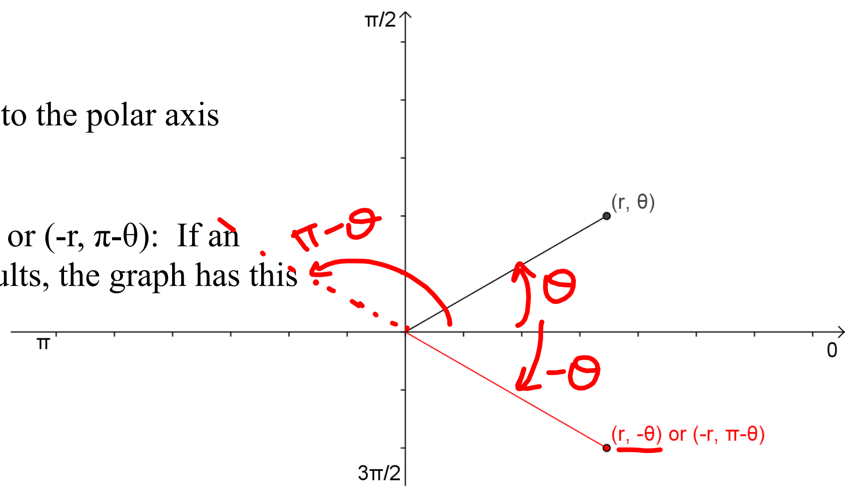
(y-axis)

Replace (r,θ) with $(r,\pi-\theta)$ or $(-r, -\theta)$: If an equivalent equation results, the graph has this type of symmetry.

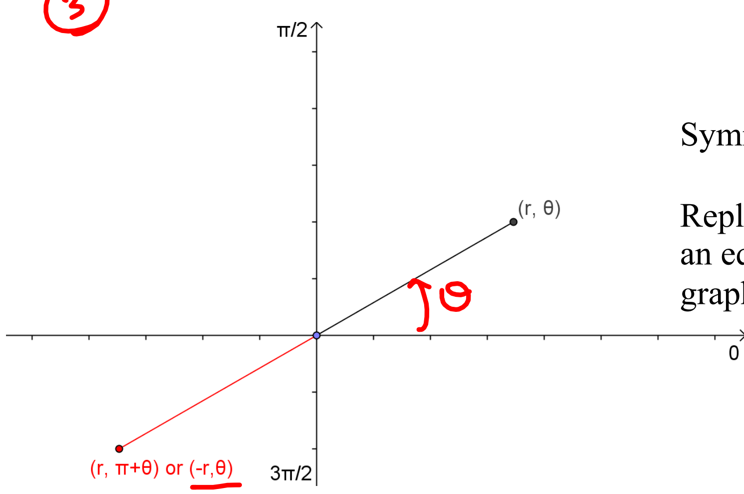
2

Symmetry with respect to the polar axis ($\theta=0$): (x-axis)

Replace (r,θ) with $(r,-\theta)$ or $(-r, \pi-\theta)$: If an equivalent equation results, the graph has this type of symmetry.



3



Symmetry with respect to the pole (origin)

Replace (r,θ) with $(-r,\theta)$ or $(r, \pi+\theta)$: If an equivalent equation results, the graph has this type of symmetry.

If a polar equation passes a symmetry test, then its graph definitely exhibits that symmetry. However, if a polar equation fails a symmetry test, then its graph may or may not have that kind of symmetry.

Zeros and maximum *r*-values

Other helpful tools in graphing polar equations are knowing the values for θ for which |*r* | is maximum and those for which $r = 0$.

Ex 2: Graph $r = \frac{1}{2} + \cos\theta$

Symmetry: ① replace (r,θ) w/ $(-r,-\theta)$: $-r = \frac{1}{2} + \cos(-\theta)$
$-r = \frac{1}{2} + \cos(\theta)$
$\not\Leftrightarrow r = \frac{1}{2} + \cos\theta$

② replace (r,θ) w/ $(r,-\theta)$: $r = \frac{1}{2} + \cos(-\theta)$
symmetry wrt $\theta = 0$ (x-axis) ✓ $r = \frac{1}{2} + \cos\theta$

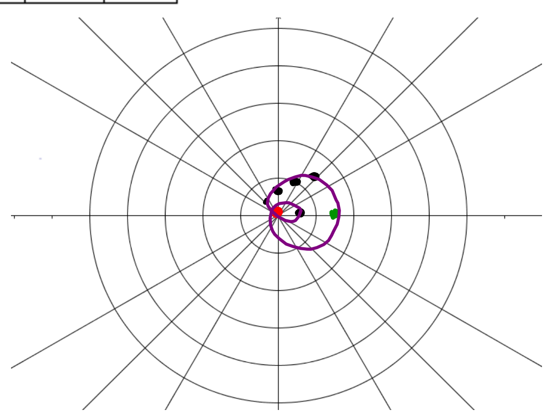
③ replace (r,θ) w/ $(-r,\theta)$: $-r = \frac{1}{2} + \cos\theta$
$\not\Leftrightarrow r = \frac{1}{2} + \cos\theta$

|*r* | maximum: $r = \frac{1}{2} + \cos\theta$
max when $\cos\theta = 1 \Rightarrow r$ max value $= 1\frac{1}{2} = \frac{3}{2}$ when $\theta = 0, 2\pi, \ldots$

Zero of *r* :
$0 = \frac{1}{2} + \cos\theta \Leftrightarrow \cos\theta = \frac{-1}{2} \Leftrightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
r	$\frac{3}{2}$	$\frac{1+\sqrt{2}}{2}$	1	$\frac{1}{2}$	0	$\frac{1-\sqrt{2}}{2}$	$\frac{-1}{2}$

$r = \frac{1}{2} + \cos\theta$



Limaçon

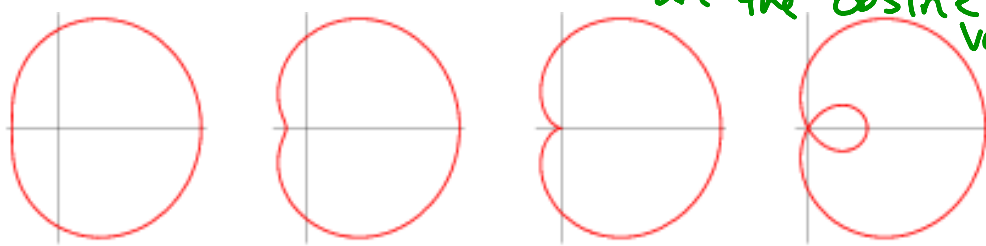
Limaçons

$$\boxed{\begin{array}{l} r = a \pm b\cos\theta \\ \\ r = a \pm b\sin\theta \end{array}}$$

a, b constants

$a > 0, \;\; b > 0$

(all these picture examples are the cosine varieties)



$\dfrac{a}{b} \geq 2$	$1 < \dfrac{a}{b} < 2$	$\dfrac{a}{b} = 1$	$\dfrac{a}{b} < 1$
Convex limaçon	Dimpled limaçon	Cardioid -always passes through pole	Limaçon with inner loop

Ex 3: Graph $r = 3\sin 2\theta$

Symmetry: try replacing (r, θ) w/ $(-r, -\theta)$

$-r = 3\sin(-2\theta)$

$-r = -3\sin(2\theta)$

$r = 3\sin(2\theta)$ ✓

symmetry wrt $\theta = \frac{\pi}{2}$ (y-axis)

r | maximum:

r is max when $\sin(2\theta) = \pm 1 \Rightarrow 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

$(3, \frac{\pi}{4})$ $(-3, \frac{3\pi}{4})$

Zero of r :

$0 = 3\sin(2\theta)$

$\sin(2\theta) = 0 \Rightarrow 2\theta = 0, \pi$

$\theta = 0, \frac{\pi}{2}$

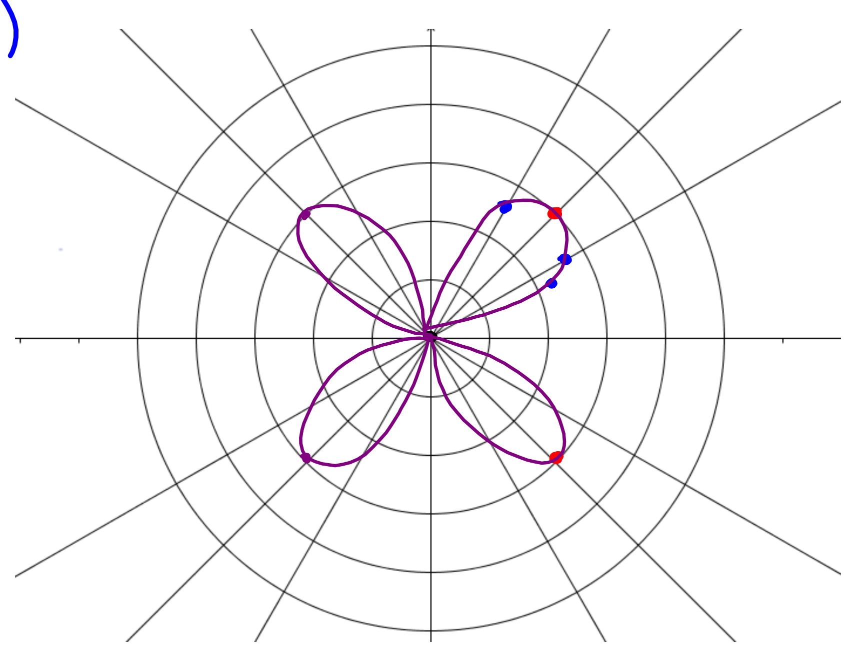
θ	0	$\pi/8$	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
r	0	$\frac{3\sqrt{2}}{2}$	$\frac{3\sqrt{3}}{2}$	3	$\frac{3\sqrt{3}}{2}$	0

$r = 3\sin(2\theta)$

$\frac{3\sqrt{2}}{2} \simeq 2.12$

$\frac{3\sqrt{3}}{2} \simeq 2.6$

$(-3, \frac{3\pi}{4})$



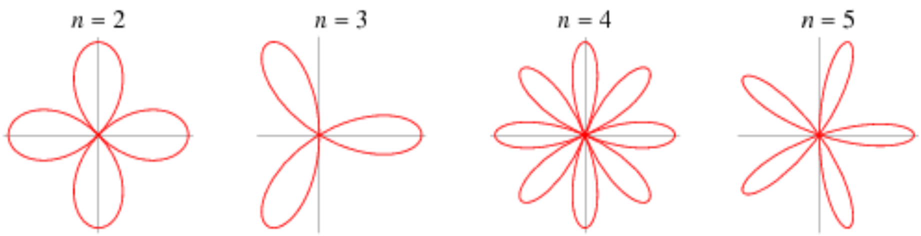
Roses

$$\boxed{\begin{array}{l} r = a \sin(n\,\theta), \\ \\ r = a \cos(n\,\theta). \end{array}}$$

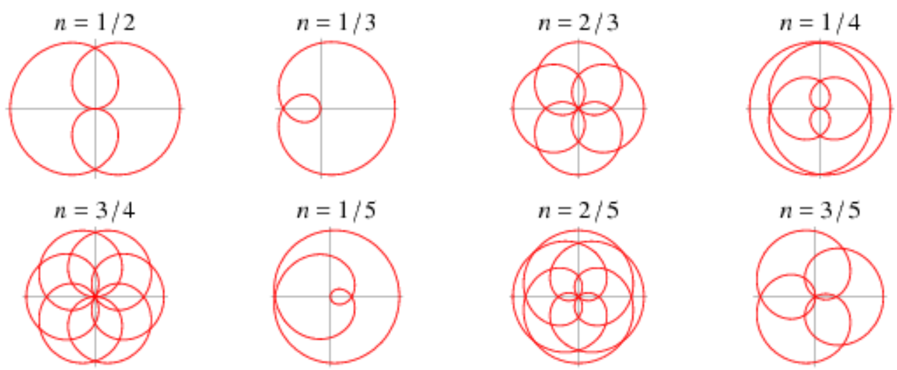
or

a constant
n constant

If n is odd, the rose is n-petalled. If n is even, the rose is $2n$-petalled.



No reason to limit ourselves to n integer:



Or even rational: