Math 1060 ~ Trigonometry

17 Polar Coordinates and Equations

Learning Objectives

In this section you will:

- Graph points in polar coordinates.
- Convert points in polar coordinates to rectangular coordinates and vice versa.
- Convert between rectangular and polar equations.
Rectangular Coordinates: \((x, y)\)

- pt given as an ordered pair
- tells how far over (horizontally) to go and how far up/down to go (vertically)

Polar Coordinates: \((r, \theta)\)

- pts are still given as ordered pairs
- tells distance from origin to travel along the \(\theta\) radial line
In fact:

\((r, \theta)\) has infinitely many representations:

\((r, \theta + 2n\pi)\) and \((- r, \theta + (2n+1)\pi)\), where \(n\) is any integer.
How do we translate between Cartesian and polar coordinates?

Polar to Cartesian: 

\[
given \ (r, \theta), \ what \ is \ (x,y) ?
\]

\[
x = r \cos \theta, \quad y = r \sin \theta
\]

\[
\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r}
\]

Ex1: Convert \((-4, \ 2\pi/3)\) to Cartesian coordinates. 

\[
x = r \cos \theta = -4 \cos \left(\frac{2\pi}{3}\right) = -4 \left(\frac{1}{2}\right) = 2
\]

\[
y = r \sin \theta = -4 \sin \left(\frac{2\pi}{3}\right) = -4 \left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}
\]
How do we translate between Cartesian and polar coordinates?

Cartesian to polar:

Ex 2: Convert (-2, 2) to polar coordinates.

\[ x^2 + y^2 = r^2 \]
\[ (-2)^2 + 2^2 = r^2 \]
\[ r^2 = 8 \]
\[ r = \pm \sqrt{8} = \pm 2\sqrt{2} \]
\[ \tan \theta = \frac{2}{-2} = -1 \]
\[ \theta = -\frac{\pi}{4} + n\pi \]

\((2\sqrt{2}, \frac{3\pi}{4})\) \quad \((-2\sqrt{2}, \frac{7\pi}{4})\) \quad \((2\sqrt{2}, \frac{5\pi}{4})\)
We can convert equations, too!

Ex 3:
(a) Convert \( x^2 - 3x = l + xy \) into polar coordinates.

\[
\begin{align*}
    x &= r \cos \theta \\
    y &= r \sin \theta \\
    r^2 \cos^2 \theta - 3r \cos \theta &= 1 + (r \cos \theta)(r \sin \theta) \\
    r^2 \cos^2 \theta - 3r \cos \theta &= 1 + r^2 \sin \theta \cos \theta \\
    r^2 \cos^2 \theta - 3r \cos \theta - r^2 \sin \theta \cos \theta &= 1
\end{align*}
\]

(b) Convert \( r = -2 \cos \theta \) into Cartesian coordinates.

\[
\begin{align*}
    r^2 &= -2r \cos \theta \\
    x^2 + y^2 &= -2x \\
    x^2 + 2x + y^2 &= 0 \\
    (x^2 + 2x + 1) - 1 + y^2 &= 0 \\
    (x+1)^2 + y^2 &= 1 \quad \text{(Circle of radius 1, centered at (1,0))}
\end{align*}
\]