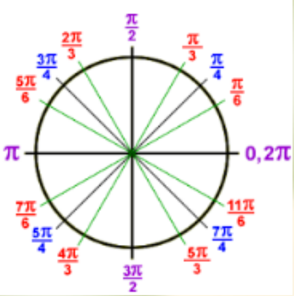
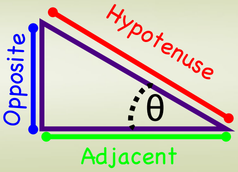
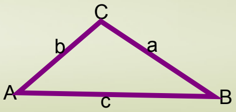
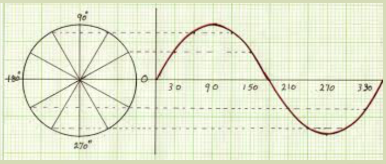








$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2\sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Math 1060 ~ Trigonometry

17 Polar Coordinates and Equations

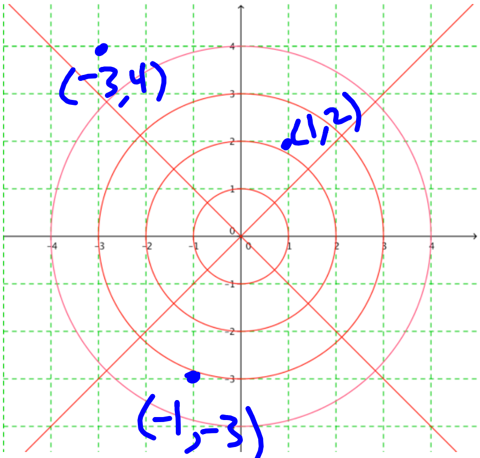
Learning Objectives

In this section you will:

- Graph points in polar coordinates.
- Convert points in polar coordinates to rectangular coordinates and vice versa.
- Convert between rectangular and polar equations.

Rectangular Coordinates: *(x,y)*

- pt given as an ordered pair
- tells how far over (horizontally) to go and how far up/down to go (vertically)



ex

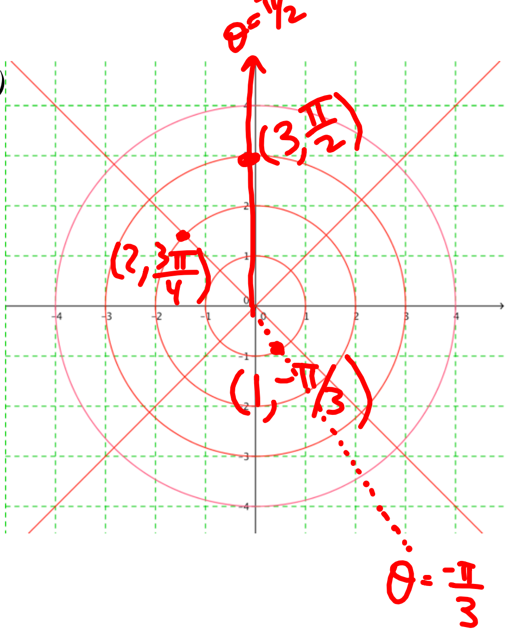
(1, 2)

(-3, 4)

(-1, -3)

Polar Coordinates: *(r,θ)*

- pts are still given as ordered pairs
- tells distance from origin to travel, along the θ radial line



ex

$(3, \frac{\pi}{2})$

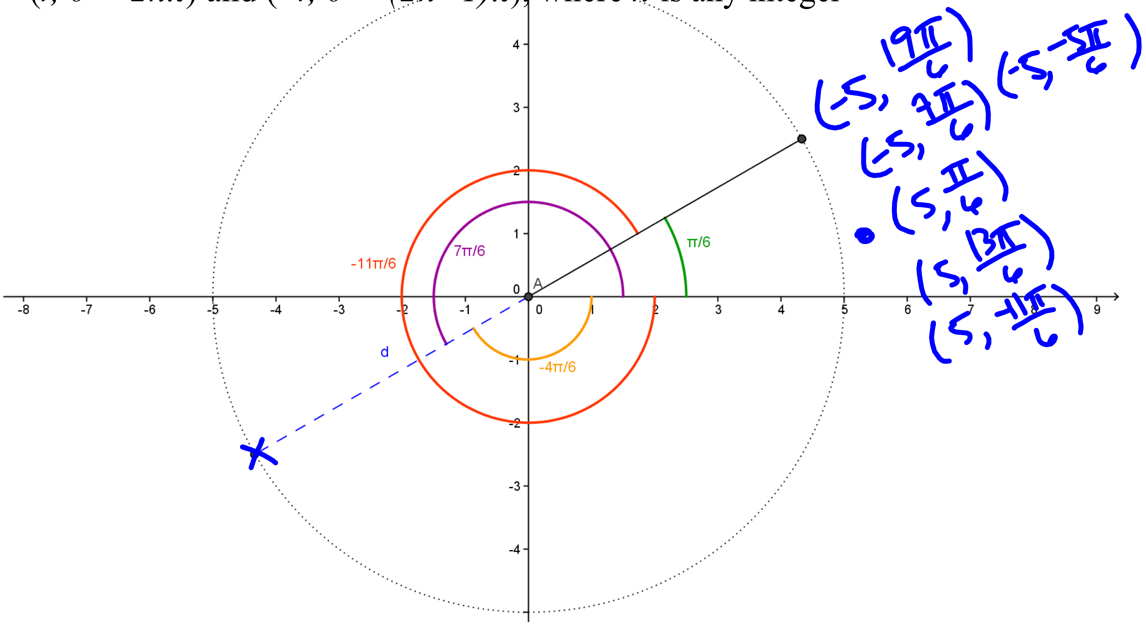
$(2, \frac{3\pi}{4})$

$(1, -\pi/3)$

In fact:

(r, θ) has infinitely many representations:

$(r, \theta + 2n\pi)$ and $(-r, \theta + (2n+1)\pi)$, where n is any integer



$(-5, \frac{19\pi}{6})$

$(-5, \frac{7\pi}{6})$ $(-5, \frac{-5\pi}{6})$

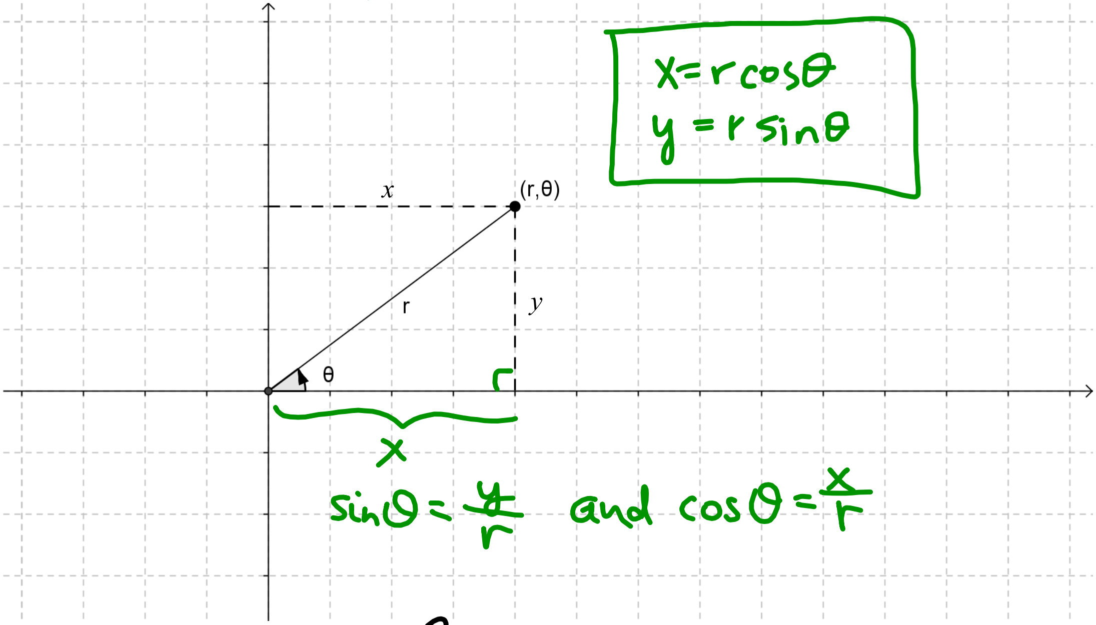
• $(5, \frac{\pi}{6})$

$(5, \frac{13\pi}{6})$

$(5, \frac{-11\pi}{6})$

How do we translate between Cartesian and polar coordinates?

Polar to Cartesian: given (r,θ), what is (x,y)?

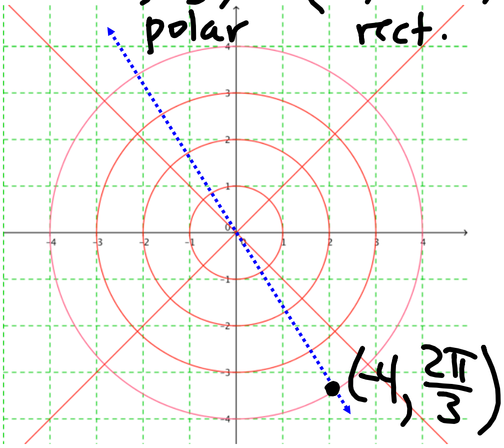


$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

$\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$

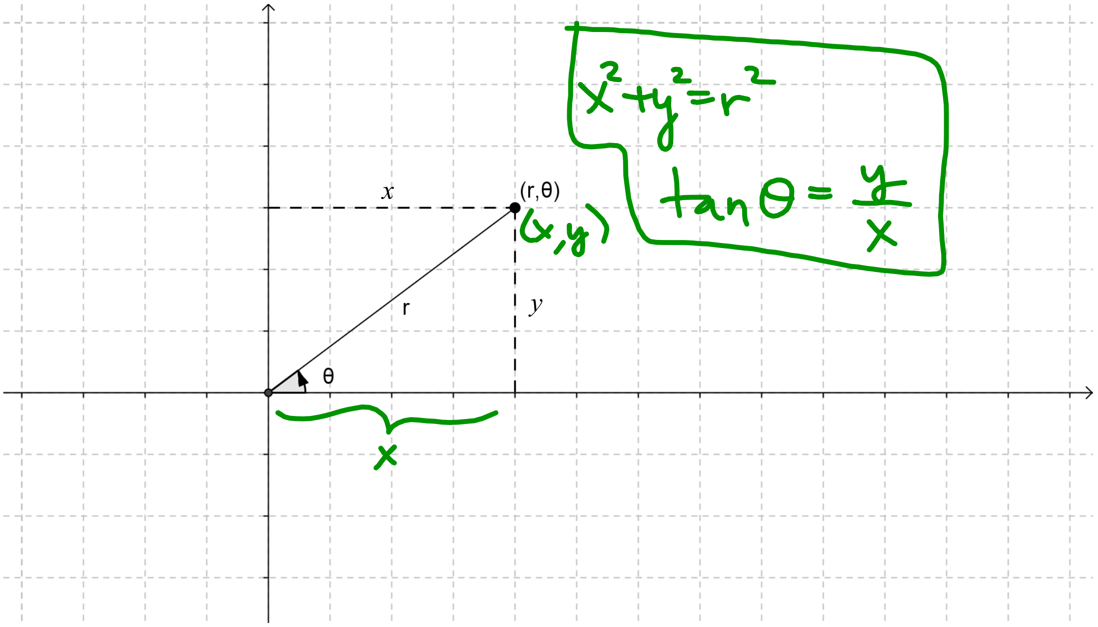
Ex1: Convert $\overset{r}{-4}$, $\overset{\theta}{2\pi/3}$) to Cartesian coordinates. $\underset{\text{polar}}{\left(-4, \frac{2\pi}{3}\right)} = \underset{\text{rect.}}{\left(2, -2\sqrt{3}\right)}$

$$x = r\cos\theta = -4\cos\left(\frac{2\pi}{3}\right)$$
$$= -4\left(\frac{-1}{2}\right) = 2$$
$$y = r\sin\theta = -4\sin\left(\frac{2\pi}{3}\right)$$
$$= -4\left(\sqrt{3}/2\right) = -2\sqrt{3}$$



How do we translate between Cartesian and polar coordinates?

Cartesian to polar:

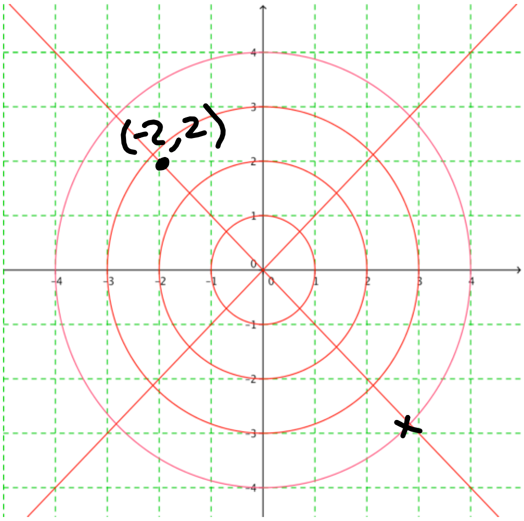


$$\boxed{\begin{array}{l} x^2+y^2=r^2 \\ \tan\theta = \dfrac{y}{x} \end{array}}$$

Ex 2: Convert $\overset{(x,y)}{(-2,\ 2)}$ to polar coordinates.

$$x^2+y^2=r^2$$
$$(-2)^2+2^2=r^2$$
$$8=r^2$$
$$r=\pm\sqrt{8}=\pm 2\sqrt{2}$$

$$\tan\theta=\frac{y}{x}=\frac{2}{-2}=-1$$
$$\theta=\frac{-\pi}{4}+n\pi$$



$$\left(2\sqrt{2},\ \frac{3\pi}{4}\right) \qquad \left(-2\sqrt{2},\ \frac{-\pi}{4}\right) \qquad \left(2\sqrt{2},\ \frac{-5\pi}{4}\right)$$

We can convert equations, too!

Ex 3:
(a) Convert x^2-$3x$=1+xy into polar coordinates.

$x = r\cos\theta$

$y = r\sin\theta$

$r^2\cos^2\theta - 3r\cos\theta = 1 + (r\cos\theta)(r\sin\theta)$

$r^2\cos^2\theta - 3r\cos\theta = 1 + r^2\sin\theta\cos\theta$

$r^2\cos^2\theta - 3r\cos\theta - r^2\sin\theta\cos\theta = 1$

(b) Convert r=$-2\cos\theta$ into Cartesian coordinates.

$r^2 = -2r\cos\theta$

$x^2+y^2=r^2$ $\quad \tan\theta = \frac{y}{x}$

$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2=-2x$

$x^2+2x+y^2=0$

$(x^2+2x+1)-1+y^2=0$

$(x+1)^2+y^2=1$ (Circle of radius 1, centered at $(-1,0)$)