

Math 1060 ~ Trigonometry

10 Verifying Trigonometric Identities

Learning Objectives

In this section you will:

- Learn and use the even/odd identities in simplifying trigonometric expressions and verifying identities.
- Learn the sum and difference identities for cosine, sine and tangent.
- Use the sum and difference identities to find values of trigonometric functions.
- Use the sum and difference identities in verifying trigonometric identities.
- Learn and apply the cofunction identities.

$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2 \sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab \cos C$

Identities are useful in simplifying expressions and computing values for some of the less-familiar angles on the Unit Circle. In this video, we will familiarize ourselves with some important identities and use them to compute values, simplify expressions and verify other identities.

| The Even/Odd Identities: For all applicable angles θ , | | |
|---|----------------------------------|-----------------------------------|
| • $\sin(-\theta) = -\sin(\theta)$ | • $\cos(-\theta) = \cos(\theta)$ | • $\tan(-\theta) = -\tan(\theta)$ |
| • $\csc(-\theta) = -\csc(\theta)$ | • $\sec(-\theta) = \sec(\theta)$ | • $\cot(-\theta) = -\cot(\theta)$ |

Ex 1: Simplify these expressions.

- a) $\sin(-x)\cos(-x)\sec(-x)$ b) $-\cot(-x)\tan(x)$

Let's play a little True False game.

Ex 2: Identify these as True or False:

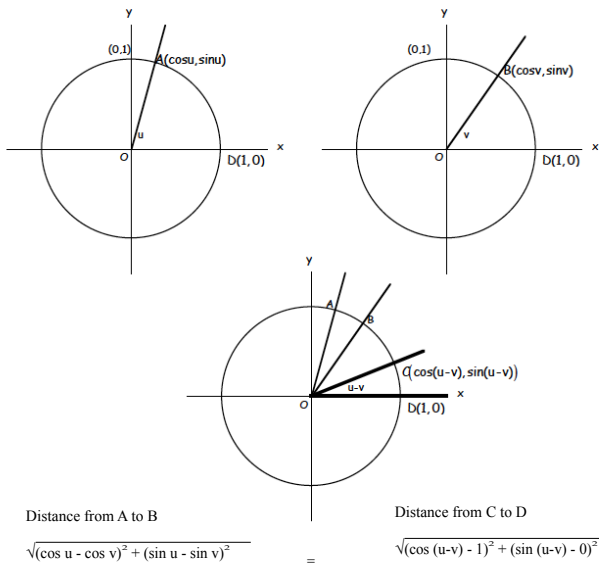
- a) $5(c+d) = 5c+5d$
- b) $\frac{a+b}{2} = \frac{a}{2} + \frac{b}{2}$
- c) $(x+y)^2 = x^2 + y^2$
- d) $\sqrt{p+q} = \sqrt{p} + \sqrt{q}$
- e) $\sin(u+v) = \sin u + \sin v$

We can tell by inspection that $\sin 30^\circ + \sin 60^\circ \neq \sin 90^\circ$.

So, we need a set of sum/difference identities for the sine.

Sum and Difference Identities: For all applicable angles α and β ,

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
- $\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$



Sum and Difference Identities for Sine. For all angles α and β ,

- $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
- $\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

Ex 3: Determine these exact values of $\sin 75^\circ$ in two ways.

a) $\sin(75^\circ) = \sin(30^\circ + 45^\circ)$

b) $\sin(75^\circ) = \sin(120^\circ - 45^\circ)$

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Ex 4: Use the sum/difference identities to find the exact values of these.

a) $\cos \frac{5\pi}{12}$

b) $\sin \left(-\frac{7\pi}{12} \right)$

Ex 5: $\tan 165^\circ$

Ex 6: Verify this cofunction identity.

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$