Reminders About a Function and Its Inverse

The inverse of a function, $f(x)$, is written $f^{-1}(x)$ (read $f$-inverse).

The $^{-1}$ is NOT an exponent.
The original function must be 1-to-1.
The graph $y = f^{-1}(x)$ (the inverse function) is a reflection of $y = f(x)$ across the line $y = x$.
An $(a,b)$ pair on the function becomes a $(b,a)$ pair on the inverse.
$f^{-1}(f(x)) = x$ for every $x$ in the domain of $f^{-1}(x)$, and vice versa.
The domain of $f^{-1}(x)$ is the range of $f(x)$ and vice versa.
Some questions about a familiar function:

What is the square root of 4?

What number(s) can I square to get 4?

\[ x^2 = 4, \text{ so } x = ? \]

\[ \sqrt{4} = ? \]

What is the principal square root of 4?

If \( x = -3 \), then \( \sqrt{x^2} = \)

If \( x = -3 \), then \( (\sqrt{x})^2 = \)

so, \( \sqrt{x^2} = \)

and \( (\sqrt{x})^2 = \)

The original function must be \( f \rightarrow \).

The graph \( y = f^{-1}(x) \) (the inverse function) is a reflection of \( y = f(x) \) across the line \( y = x \).

An \((a,b)\) pair on the function becomes a \((b,a)\) pair on the inverse.

\( f(f^{-1}(x)) = x \) for every \( x \) in the domain of \( f^{-1}(x) \), and vice versa.

The domain of \( f^{-1}(x) \) is the range of \( f(x) \) and vice versa.
As we determine inverses of our trigonometric functions, this is why

\( \sin x = 0.5 \) has many solutions for \( x \), and \( \sin^{-1}(0.5) \rightarrow ? \) has only one answer.