Extra ~ Review of Inverse Functions

You will learn to:

• Determine whether a function has an inverse.
• Find and verify the inverse function if there is one.
• Sketch a function and its inverse.
**Reminders About a Function and Its Inverse**

The inverse of a function, \( f(x) \), is written \( f^{-1}(x) \) (read f-inverse).

- The \(-1\) is NOT an exponent.
- The original function must be **1-to-1**.
  - The graph \( y = f^{-1}(x) \) (the inverse function) is a reflection of \( y = f(x) \) across the line \( y = x \).
- An \((a, b)\) pair on the function becomes a \((b, a)\) pair on the inverse.
  - \(f(f^{-1}(x)) = x\) for every \(x\) in the domain of \(f^{-1}(x)\), and vice versa.
- The domain of \(f^{-1}(x)\) is the range of \(f(x)\) and vice versa.

![Graph](image)
Some questions about a familiar function:

What is the square root of 4? \( \sqrt{4} = 2 \)

What number(s) can I square to get 4?

- \( x^2 = 4 \), so \( x = ? \) ± 2
- \( \sqrt{4} = \) 2

What is the \textit{principal} square root of 4?

\[
\begin{align*}
\text{If } x &= -3, \text{ then } \sqrt{x^2} &= \sqrt{(-3)^2} = \sqrt{9} = 3 \\ 
\text{If } x &= -3, \text{ then } (\sqrt{x})^2 &= (\sqrt{-3})^2 \quad \text{not possible} \\
\text{so, } \sqrt{x^2} &= 1 \times 1 \\
\text{and } (\sqrt{x})^2 &= x \quad \text{(assuming that } x \geq 0)\\
\end{align*}
\]

Squaring undoes \textit{sq.root} but \textit{sq.root} does not undo squaring.

\[ \text{moral of the story: } \sqrt{x^2} = |x| \neq x \]
\[ \text{i.e. square root fn and square fn do NOT always undo each other!} \]
mostly, we choose scenario ① to be the inverse curve.

Check (for ①) \( f(x) = x^2, \ x \geq 0 \), \( f^{-1}(x) = \sqrt{x} \)

\[
f(f^{-1}(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x
\]

(for ②) \( f(x) = x^2, \ x \leq 0 \), \( f^{-1}(x) = -\sqrt{x} \)

\[
f(f^{-1}(x)) = f(-\sqrt{x}) = (-\sqrt{x})^2 = x
\]
As we determine inverses of our trigonometric functions, this is why

\( \sin x = 0.5 \) has many solutions for \( x \), and \( \sin^{-1}(0.5) = ? \) has only one answer.

\[ y = \frac{1}{2} \]

\[ y = \sin x \]

\[ \Rightarrow y = \sin x \text{ does not pass horizontal line test} \]

\[ \Rightarrow \sin x = f(x) \text{ is NOT invertible unless we restrict domain of } f(x) = \sin x \]