3.6 ~ Introduction to polar coordinates

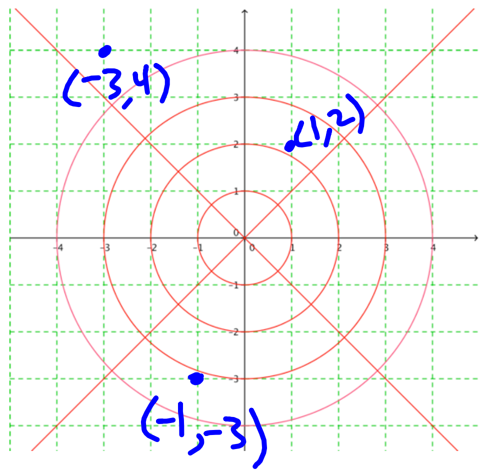
In this lesson you will:

- Learn what polar coordinates are.
- Convert between polar coordinates and rectangular coordinates.
- Convert between polar and rectangular equations.

Rectangular Coordinates

(x,y)

- pt given as an ordered pair
- tells how far over (horizontally) to go and how far up/down to go (vertically)



ex

$(1,2)$

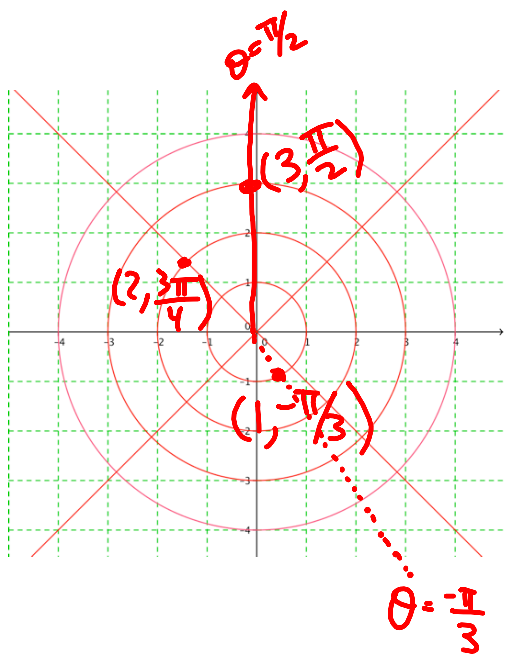
$(-3,4)$

$(-1,-3)$

Polar Coordinates

(r,θ)

- pts are still given as ordered pairs
- tells distance from origin to travel, along the θ radial line



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$(3,\frac{\pi}{2})$

$(2,\frac{3\pi}{4})$

$(1,-\pi/3)$

In fact:

(r, θ) has infinitely many representations:

$(r, \theta + 2n\pi)$ and $(-r, \theta + (2n+1)\pi)$, where n is any integer

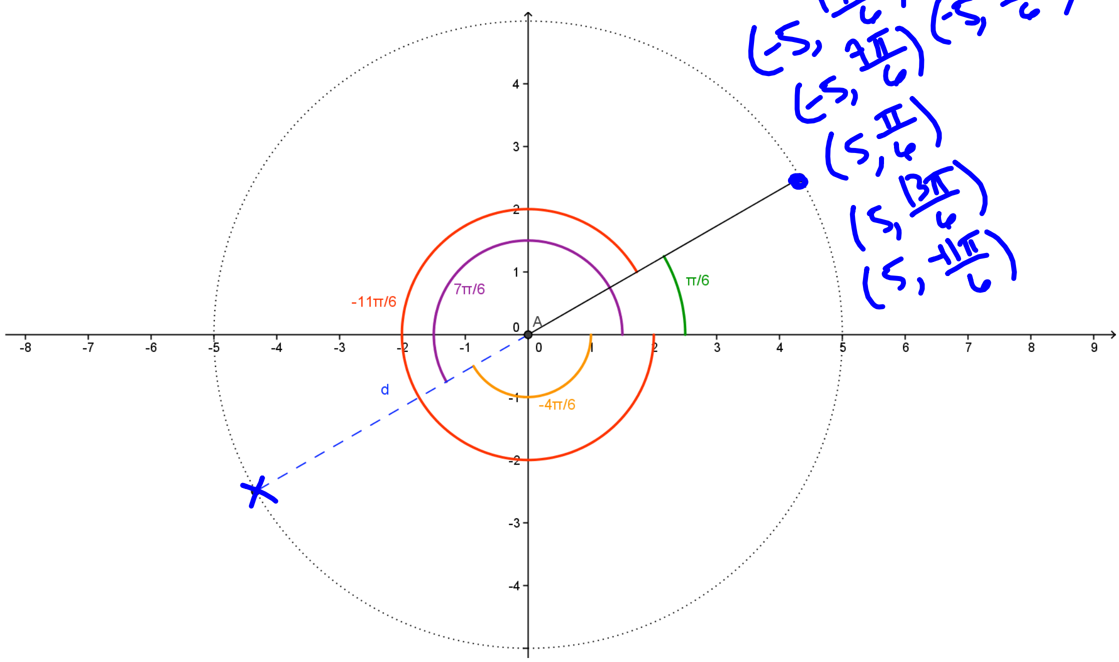
$(-5, \frac{19\pi}{6})$ $(-5, -\frac{5\pi}{6})$

$(-5, \frac{7\pi}{6})$

$(5, \frac{\pi}{6})$

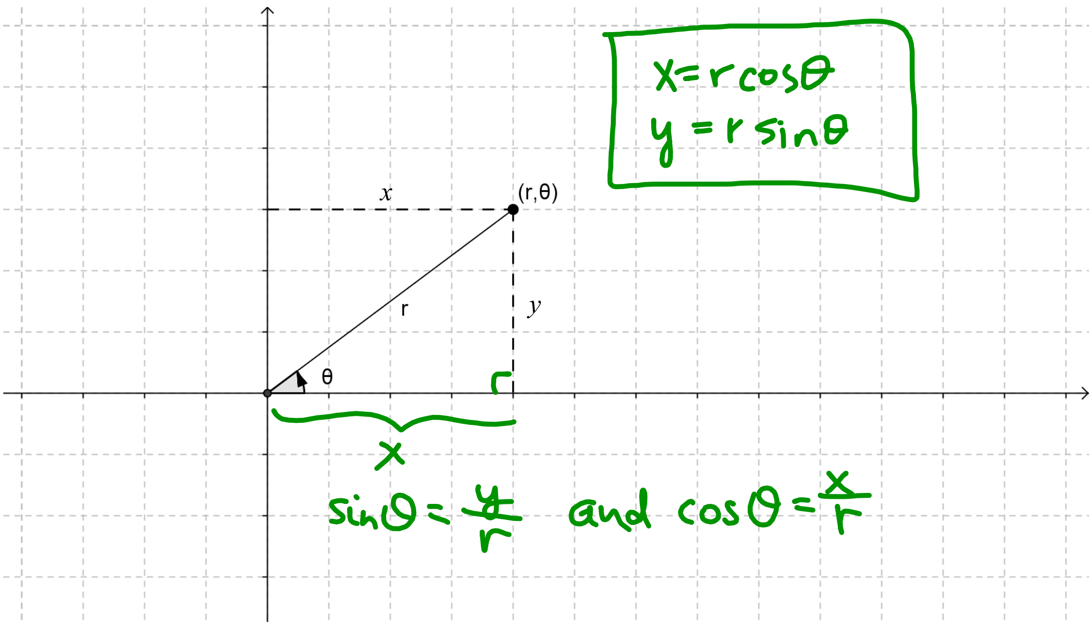
$(5, \frac{13\pi}{6})$

$(5, -\frac{11\pi}{6})$



How do we translate between Cartesian and polar coordinates?

Polar to Cartesian given (r,θ), what is (x,y)?



$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

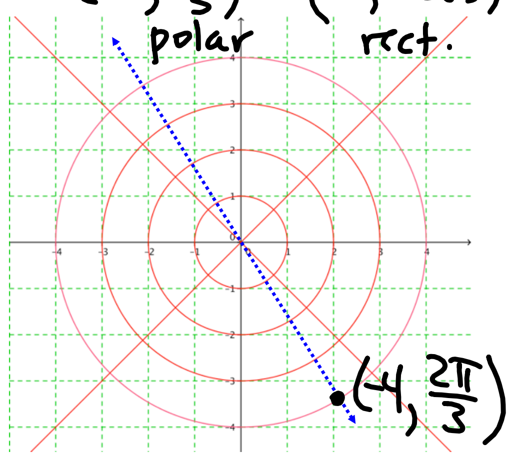
$\underbrace{\qquad\qquad}_{x}$

$\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$

Example 1: Convert $\overset{r}{(-4}, \overset{\theta}{2\pi/3)}$ to Cartesian coordinates

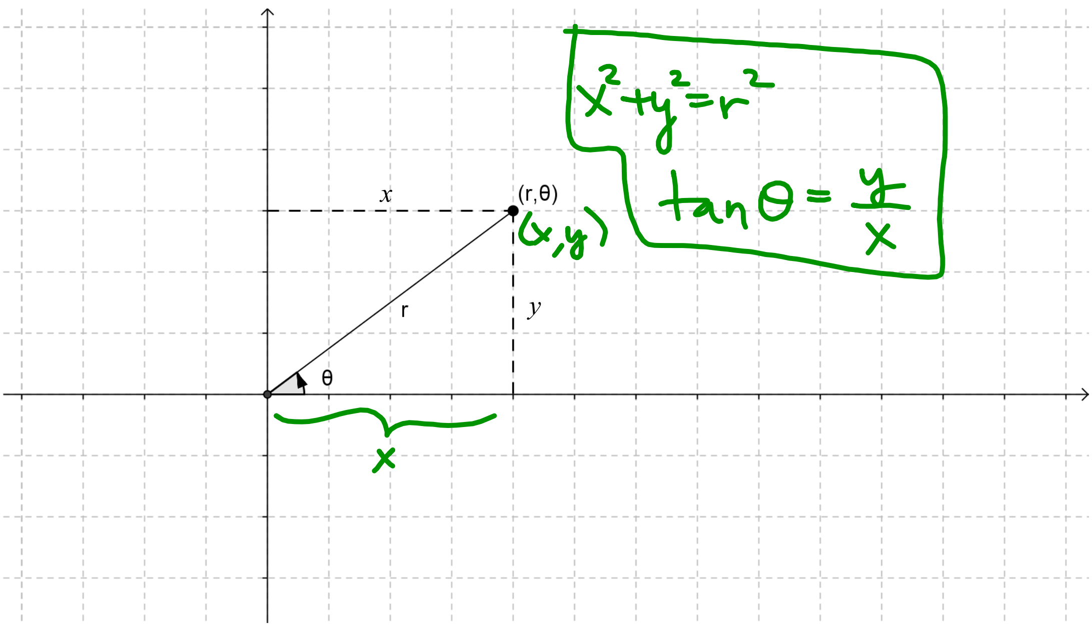
$$\underset{\text{polar}}{\left(-4, \frac{2\pi}{3}\right)} = \underset{\text{rect.}}{\left(2, -2\sqrt{3}\right)}$$

$$x = r\cos\theta = -4\cos\left(\frac{2\pi}{3}\right)$$
$$= -4\left(\frac{-1}{2}\right) = 2$$
$$y = r\sin\theta = -4\sin\left(\frac{2\pi}{3}\right)$$
$$= -4\left(\sqrt{3}/2\right) = -2\sqrt{3}$$



How do we translate between Cartesian and polar coordinates?

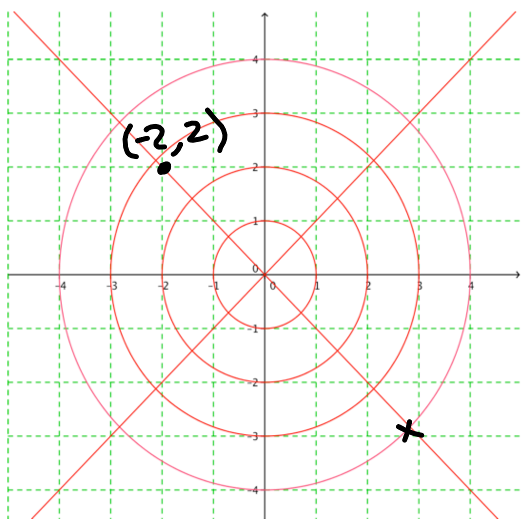
Cartesian to polar



$$\boxed{\begin{gathered} x^2+y^2=r^2 \\ \tan\theta = \frac{y}{x} \end{gathered}}$$

(x,y)

Example 2: Convert *(-2, 2)* to polar coordinates



$$x^2+y^2=r^2$$

$$(-2)^2+2^2=r^2$$

$$8=r^2$$

$$r=\pm\sqrt{8}=\pm 2\sqrt{2}$$

$$\tan\theta=\frac{y}{x}=\frac{2}{-2}=-1$$

$$\theta=\frac{-\pi}{4}+n\pi$$

$$\left(2\sqrt{2}, \frac{3\pi}{4}\right) \qquad \left(-2\sqrt{2}, \frac{-\pi}{4}\right) \qquad \left(2\sqrt{2}, \frac{-5\pi}{4}\right)$$

We can convert equations, too!

Example 3:

(a) Convert $x^2-3x=1+xy$ into polar coordinates.

$x=r\cos\theta$

$y=r\sin\theta$

$r^2\cos^2\theta-3r\cos\theta=1+(r\cos\theta)(r\sin\theta)$

$r^2\cos^2\theta-3r\cos\theta=1+r^2\sin\theta\cos\theta$

$r^2\cos^2\theta-3r\cos\theta-r^2\sin\theta\cos\theta=1$

(b) Convert $r=-2\cos\theta$ into Cartesian coordinates.

$x^2+y^2=r^2$ $\tan\theta=\frac{y}{x}$

$x=r\cos\theta$

$y=r\sin\theta$

$r^2=-2r\cos\theta$

$x^2+y^2=-2x$

$x^2+2x+y^2=0$

$(x^2+2x+1)-1+y^2=0$

$(x+1)^2+y^2=1$ (Circle of radius 1, centered at $(-1,0)$)