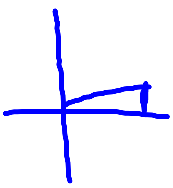
TRIG 2.3 ~ Solving Trigonometric Equations

You will learn techniques for solving equations involving trigonometric functions.

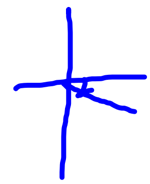
Review of inverse functions:

✓ $\sin^{-1}(1/2) = x$



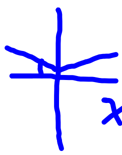
$\frac{\pi}{6} = x$

$\sin^{-1}(-1/2) = x$



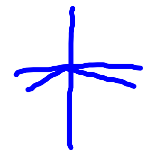
$x = -\frac{\pi}{6}$

✓ $\sin x = 1/2$



$x = \begin{cases} \frac{\pi}{6} + 2k\pi \\ \frac{5\pi}{6} + 2k\pi \end{cases}$ $(k \leftarrow \text{integers})$

$\sin x = -1/2$



$x = \begin{cases} \frac{7\pi}{6} + 2k\pi \\ \frac{11\pi}{6} + 2k\pi \end{cases}$

✓ $\sin x = 1/2,\quad \underline{0 \le x < 2\pi}$

$x = \underline{\frac{\pi}{6}, \frac{5\pi}{6}}$

$\sin x = -1/2,\quad 0 \le x < 2\pi$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\tan^{-1}(1) = \frac{\pi}{4}$

$\tan^{-1}(-1) = -\frac{\pi}{4}$

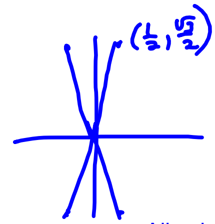
Solving a trigonometric equation

Use algebra combined with the inverse trig functions.

a) $3\cot^2 x = 1$

$\cot^2 x = \frac{1}{3}$

$\cot x = \pm \frac{1}{\sqrt{3}}$



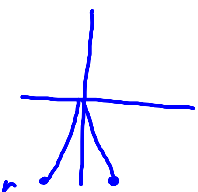
Solutions on [0,2π)

$[0,2\pi) \rightarrow x = \pi/3, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

All solutions:

$$\boxed{\begin{array}{l} x = \frac{\pi}{3} + k\pi \\ x = \frac{2\pi}{3} + k\pi \end{array}}$$

b) $2 \sin 2x = - \sqrt{3}$



$\sin 2x = \frac{-\sqrt{3}}{2}$

$2x = -\frac{\pi}{3}$ or $\rightarrow 2x = \frac{5\pi}{3} + 2k\pi$

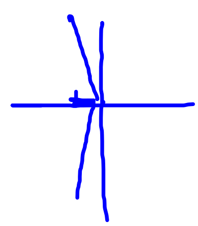
$2x = \frac{4\pi}{3} + 2k\pi$

$x = \frac{2\pi}{3} + k\pi$

$x = \frac{5\pi}{6} + k\pi$

4 answers on $[0,2\pi)$

c) sec (x/2) = -2



$\cos \frac{x}{2} = -\frac{1}{2}$

$\frac{x}{2} = \frac{2\pi}{3} + 2k\pi$

$\frac{x}{2} = \frac{4\pi}{3} + 2k\pi$

$x = \frac{4\pi}{3} + 4k\pi$

$= \frac{8\pi}{3} + 4k\pi$

$[0,2\pi)$

$\Rightarrow \frac{4\pi}{3}$

Some algebra may be required.
You may need to multiply or factor...

a) Solve for x on the interval [0,2π)

sin x cos x - cos x = 0

$\cos x(\sin x - 1) = 0$

$\cos x = 0 \qquad \sin x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \sin x = 1$

$\frac{\pi}{2}$

You may need to put the expression in terms of the same function using the identities.

$\cos^2 x = 1 - \sin^2 x$

b) Solve for x on the interval [0,2π)

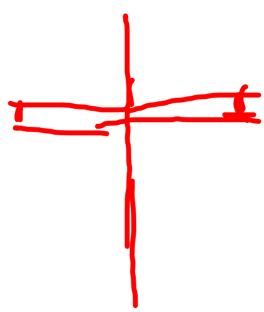
sin x + 2 cos²x - 2 = 0

$\sin x + 2(1 - \sin^2 x) - 2 = 0$

$\sin x + 2 - 2\sin^2 x - 2 = 0$

$\sin x - 2\sin^2 x = 0$

$\sin x(1 - 2\sin x) = 0$



$\sin x = 0 \qquad \sin x = \frac{1}{2}$

$x = 0, \pi \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x \in \{0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}\}$

Using inverse trig functions to state a solution.

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

State the solutions to this equation:

$\sec^2 x - 2\tan x = 4$

$$\tan^2 x + 1 - 2\tan x - 4 = 0$$

$$\tan^2 x - 2\tan x - 3 = 0$$

$$(\tan x - 3)(\tan x + 1) = 0$$

$$\tan x = 3 \qquad \tan x = -1$$

$$x = \tan^{-1}(3) + k\pi \approx 1.25 + k\pi$$

$$x = -\frac{\pi}{4} + k\pi$$

Common error:

if $ab=0$ $a=0$ or $b=0$

if $ab=1$ no information

$\sin x(\cos x-1)=2$

Cannot <u>do</u>

$\sin x\cos x=\sin x$

$\sin x\cos x-\sin x=0$

$\sin x(\cos x-1)=0$