2.1 Trigonometry ~ Fundamental Identities

*You will recognize and write the fundamental identities.

* Use the fundamental identities to evaluate, simplify and rewrite trigonometric expressions.

Terminology

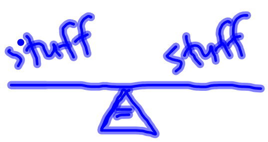
Expression $\Rightarrow$ fragment of a sentence

ex $3x - 4x + 5 + 7$

$= -x + 12$

Equation $\Rightarrow$ complete sentence

ex $3x - 4x = 5 + 7$



$-x = 12$

$x = -12$

Identity $\Rightarrow$ particular kind of equation

$\sin^2 x + \cos^2 x = 1$

Identities we already know:

Reciprocal identities

$\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$

Quotient identities

$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean identities

① $\boxed{\sin^2 x + \cos^2 x = 1}$

② $\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$1 + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$\boxed{1 + \cot^2 x = \csc^2 x}$

③ $\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

③ $\boxed{\tan^2 x + 1 = \sec^2 x}$

Cofunction identities

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

Examples of using identities:

a. To solve a problem:
sec u = -5/4 and tan u > 0 Find sin u.

Q3

(cos u negative, sin u negative)

$\cos u = \frac{-4}{5}$

$\left(\frac{-4}{5}\right)^2 + \sin^2 u = 1$

$\frac{16}{25} + \sin^2 u = 1$

$\sqrt{\sin^2 u} = \sqrt{\frac{9}{25}}$

$\sin u = \frac{\pm 3}{5}$

$\boxed{\sin u = \frac{-3}{5}}$

b. To simplify an expression:

$$\frac{1}{\underbrace{\tan^2 x + 1}_{\text{expression}}} = \frac{1}{\sec^2 x} = \cos^2 x$$

$\Rightarrow \frac{1}{\tan^2 x + 1} = \cos^2 x$ identity

c. Simplify $\cos t\,(1+\tan^2 t)$

$= \cos t\,(\sec^2 t)$

$= \cancel{\cos t}\left(\frac{1}{\cos^{\cancel{2}} t}\right) = \frac{1}{\cos t} = \sec t$

d. Use algebra on trigonometric expressions

Factor: $\sin^2 x \sec^2 x - \sin^2 x =$

$\sin^2 x\,(\sec^2 x - 1)$

$= \sin^2 x \tan^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

e. Simplify: $\dfrac{\cos^2 x - 4}{\cos x - 2}$

$$= \frac{\cancel{(\cos x - 2)}(\cos x + 2)}{\cancel{(\cos x - 2)}} = \cos x + 2$$

f. Multiply: (3 - sin x) (3 + sin x)

$$= 9 + \cancel{3\sin x} - \cancel{3\sin x} - \sin^2 x$$
$$= 9 - \sin^2 x$$

Try these:

a. Simplify: $\dfrac{\cot^2 x}{\csc^2 x}$

$= \dfrac{\cos^2 x}{\cancel{\sin^2 x}\left(\dfrac{1}{\cancel{\sin^2 x}}\right)}$

$= \cos^2 x$

b. Simplify: $\tan x - \dfrac{\sec^2 x}{\tan x}$

$= \tan x\left(\dfrac{\tan x}{\tan x}\right) - \dfrac{\sec^2 x}{\tan x}$

$= \dfrac{\tan^2 x - \sec^2 x}{\tan x}$

$= \dfrac{-1}{\tan x} = -\cot x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x - \sec^2 x = -1$

c. Simplify: $\dfrac{\tan^2 x}{\sec x + 1}$

$$= \frac{\sec^2 x - 1}{\sec x + 1}$$

$$= \frac{(\sec x - 1)\cancel{(\sec x + 1)}}{\cancel{(\sec x + 1)}}$$

$$= \sec x - 1$$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$