Inverse Trigonometric Functions

You will learn to:

- Evaluate and graph the inverse sine function.
- Evaluate and graph the other inverse trigonometric functions.

The inverse of a function \( f(x) \) is written \( f^{-1}(x) \), pronounced \( f \) inverse of \( x \).

- The -1 is NOT an exponent.
- The original function must be 1-to-1.
- The inverse is a reflection through the line \( y = x \).
- An \( (a,b) \) pair on the function becomes a \( (b,a) \) pair on the inverse.
- The domain of \( f(x) \) is the range of \( f^{-1}(x) \) and visa versa.

Example: Inverse of \( y = x^2 \)
Inverse trig functions

$y = \sin x$

$y = \cos x$

$y = \tan x$

$y = \sin^{-1} x$

$y = \cos^{-1} x$

$y = \tan^{-1} x$
The important thing to remember is the answer to a question about an inverse function is unique and must come from a certain range.

**Arccos x must have an answer in the interval [0, π].**

**As will the arccsc x, arctan x and arccot x functions.**

**Arccos x must have an answer in the interval [0, π].**

**As will the arcsec x function.**

Try these:

\[
\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \\
\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) \quad \tan^{-1} \cdot 1 \\
\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right) \quad \sec^{-1}(-\sqrt{3}) \\
\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) \quad \tan^{-1}(-\sqrt{3}) \\
\sec^{-1}\left(-1\right) \quad \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)
\]

**Some more complex problem involving arcsin, arccos and arctan:**

*Hint: Draw a right triangle!*

a) \[\cos(\arctan(2/3))\]

b) \[\tan(\sin^{-1}(3/4))\]

c) \[\sec(\arcsin x)\]

d) \[\csc(\tan^{-1}(3x/2))\]
And a few more:

a) \( \sec(\arctan(-3/4)) \)

b) \( \cot(\sin^{-1}(-0.2)) \)

c) A plane flies at an altitude of 6 miles toward a point directly over an observer. Write the angle \( \theta \) as a function of \( x \), the horizontal distance from the observer to a point on the ground directly below the airplane.