1.7 ~ Inverse Trigonometric Functions

You will learn to:

Evaluate and graph the inverse sine function.
Evaluate and graph the other inverse trigonometric functions.
The inverse of a function \( f(x) \) is written \( f^{-1}(x) \), pronounced \( f \) inverse of \( x \).

The -1 is NOT an exponent.
The original function must be 1-to-1.
The inverse is a reflection through the line \( y = x \)
An \((a,b)\) pair on the function becomes a \((b,a)\) pair on the inverse.
The domain of \( f(x) \) is the range of \( f^{-1}(x) \) and visa versa.

Example: Inverse of \( y = x^2 \)

\[
\begin{align*}
  f(x) &= x^2 \\
  f^{-1}(x) &= \sqrt{x} \\
  \sqrt{16} &= 4
\end{align*}
\]
y = \sin x

\begin{align*}
\text{Domain (D): } & [-1, 1] \\
\text{Range (R): } & [-\frac{\pi}{2}, \frac{\pi}{2}] \\
\text{Inverse: } y = \arcsin x
\end{align*}

y = \cos x

\begin{align*}
\text{Domain (D): } & [0, \pi] \\
\text{Range (R): } & [-1, 1] \\
\text{Inverse: } y = \arccos x
\end{align*}
1.7b Inverse functions

\[
y = \sin x \\
y = \cos x \\
y = \tan x \\
\]

\[
y = \sin^{-1} x \\
y = \cos^{-1} x \\
y = \tan^{-1} x \\
\]
The important thing to remember is the answer to a question about an inverse function is unique and must come from a certain range.

\[
\begin{align*}
\text{Arcsin } x & \text{ must have an answer in the interval } [-\pi/2, \pi/2]. \\
\text{As will the arccsc } x, \text{ arctan } x & \text{ and arccot } x \text{ functions.}
\end{align*}
\]

Try these:

\[
\begin{align*}
\sin^{-1}(\sqrt{3}/2) & = \frac{\pi}{3} \\
\sin^{-1}(-\sqrt{2}/2) & = -\frac{\pi}{4} \\
\cos^{-1}0 & = \frac{\pi}{2} \\
\sec^{-1}(-2/\sqrt{3}) & = \frac{5\pi}{6} \\
\cos^{-1}(\sqrt{3}/2) & = \frac{\pi}{6} \\
\tan^{-1}(-1/\sqrt{3}) & = -\frac{\pi}{6} \\
\sec^{-1}1 & = \pi \\
\cos^{-1}(\sqrt{3}/2) & = \frac{\pi}{6} \\
\tan^{-1}(-1) & = -\frac{\pi}{4} \\
\sin^{-1}(-1/2) & = -\frac{\pi}{6} \\
\sec^{-1}(-\sqrt{2}) & = \frac{3\pi}{4} \\
\cos^{-1}(-\sqrt{2}/2) & = \frac{3\pi}{4} \\
\end{align*}
\]
Some more complex problem involving arcsin, arccos and arctan:

Hint: Draw a right triangle!

a) \( \cos (\arctan (\frac{2}{3})) \)

\[
\frac{3}{\sqrt{13}}
\]

b) \( \tan (\sin^{-1}(\frac{3}{4})) \)

\[
\frac{3}{\sqrt{7}}
\]

c) \( \sec (\arcsin x) \)

\[
\frac{1}{\sqrt{1-x^2}}
\]

d) \( \csc (\tan^{-1}(\frac{3}{2})) \)

\[
\frac{\sqrt{9x^2+4}}{3x}
\]
And a few more:

a) \( \sec(\arctan(-3/4)) \)

b) \( \cot(\sin^{-1}(-0.2)) \)

c) A plane flies at an altitude of 6 miles toward a point directly over an observer. Write the angle \( \theta \) as a function of \( x \), the horizontal distance from the observer to a point on the ground directly below the airplane.
\[ \sec^{-1} (2.3) \]
\[ \cos^{-1} \left( \frac{1}{2.3} \right) \approx 1.12 \]