Trig 1.1 part 2  Angular Motion and Linear speed

You will learn to:

Determine the angular velocity of an object.
Determine the linear velocity of an object.
Find the area of a sector of a circle.
Arc Length and Speed in Circular Motion

\[ s = r \theta \]

\[ \theta = \frac{s}{r} \]

Angular speed (omega) = \( \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t} \)

Linear speed (velocity) = \( \frac{\text{arc - length}}{\text{time}} = \frac{s}{t} \)

\[ v = rw \]

- \( v \) linear velocity
- \( r \) radius
- \( \omega \) angular velocity
\[ S = r \theta \]
\[ = 9 \cdot \frac{140^\circ}{180^\circ} \cdot \pi \cdot 9 \]
\[ = 7\pi \text{m} \]

in 10 sec

\[
\text{angular vel} \quad \omega = \frac{2\pi}{10 \text{sec}} = \frac{\pi}{5} \text{ rad/ sec}
\]

\[
\text{linear vel} \quad v = r \omega = 9 \left( \frac{2\pi}{10} \right)
\]
\[ = \frac{2\pi}{10} \text{ m/sec} \]
Think about a flea on the end of a six-inch second hand on a wall clock.

There are many ways to talk about how fast he is going:

**RPM:** How many revolutions per minute does he make? \( \frac{1 \text{ rev}}{1 \text{ min}} \)

How many revolutions per hour? \( \frac{1 \text{ rev}}{\text{min}} \cdot 60 \text{ min} = 60 \text{ rev/hr} \)

\( \omicron \) What is his angular velocity in radians per hour? \( \frac{60 \text{ rev}}{\text{hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 120\pi \text{ rad/hr} \)

\( \nu \) What is his linear velocity in inches per hour?

\[ v = rw = 6 \text{ in} \cdot \frac{120\pi \text{ rad}}{\text{hr}} = 720\pi \text{ in/hr} \]

If he moves in two inches toward the center staying on the second hand, which of these will change? What will the new values be?

\[ v = 4 \text{ in} \quad \omega = \frac{120\pi \text{ rad}}{\text{hr}} \quad \nu = \frac{480\pi \text{ in}}{\text{hr}} \]
The area inside the circle and inside the angle is called a sector of a circle. It is like a slice of pie.

The area of a sector is \( A_{\text{sector}} = \frac{1}{2} r^2 \theta \)

Where \( r \) is the radius and the angle is in radians.

\[
A = \frac{\theta \text{ degrees}}{360^\circ} \cdot \pi r^2
\]

\[
= \frac{\theta}{2} \cdot \frac{180^\circ}{\pi} \cdot \pi r^2
\]

\[
= \frac{1}{2} r^2 \theta \quad \theta \text{ (radians)}
\]
Let's think of a Rainbird sprinkler watering a large field. The sprinkler takes 15 seconds to go back and forth. It is set for 150 degrees of coverage and the spray reaches out 70 feet.

\[ \theta = 150^\circ \]
\[ \theta = \frac{\pi}{180} \]
\[ \omega = \frac{\pi}{15} \text{ rad/sec} \]

What is the angular velocity of the sprinkler?

What is the linear velocity of the bird flying back and forth at the end of the water stream?

\[ v = r \omega = 70 \left( \frac{\pi}{9} \right) \text{ ft/sec} \approx 24.4 \text{ ft/sec} \]

How much area will it water?

\[ A = \frac{1}{2} r^2 \theta = \frac{1}{2} (70)^2 \cdot \frac{5\pi}{2} \approx 6414 \text{ sq ft} \]
How fast are you going when sitting in a seat on a 25-foot Ferris Wheel which makes 5 rotations each minute?

\[
\omega = \frac{5 \text{ rotations}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{10\pi}{\text{min}}
\]

\[
v = 12.5 \text{ ft} \left( \frac{10\pi}{\text{min}} \right) = 125\pi \text{ fpm} \\
\approx 393 \text{ fpm}
\]