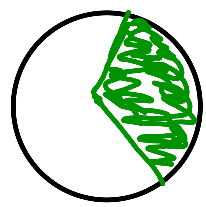
Trig 1.1 part 2 Angular Motion and Linear speed

You will learn to:

Determine the angular velocity of an object.

Determine the linear velocity of an object.

Find the area of a sector of a circle.

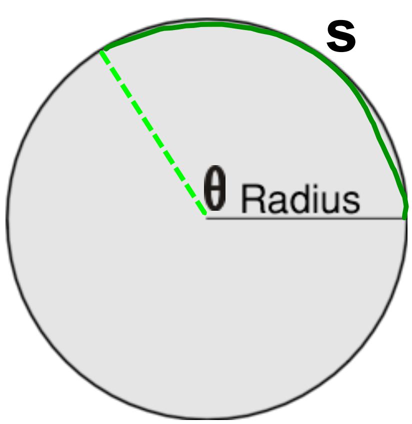


Arc Length and Speed in Circular Motion

arc radius radians

$$s = r\,\theta$$

WHY? $\theta = \frac{s}{r}$



ω Angular speed (*omega*) = $\frac{central\_angle}{time} = \frac{\theta}{t}$

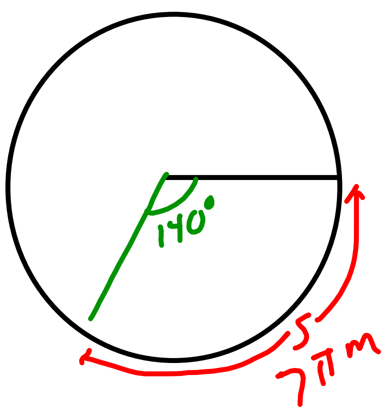
v Linear speed (*velocity*) = $\frac{arc - length}{time} = \frac{s}{t}$

$v = r\omega$

v linear velocity

r radius

ω angular velocity



$S = r\theta$

$= \cancel{9} \cdot \cancel{140}^{7}{}^\circ \cdot \dfrac{\pi}{\cancel{180}_{\cancel{9}\,1}{}^\circ}$

$= 7\pi \text{ m}$

in 10 sec

angular vel $\quad \omega = \dfrac{\frac{7\pi}{9}}{10 \text{ sec}} = \dfrac{7\pi}{90} \dfrac{\text{rad}}{\text{sec}}$

linear vel $\quad v = r\omega = 9\left(\dfrac{7\pi}{90}\right)$

$= \dfrac{7\pi}{10} \text{ m/sec}$

Think about a flea on the end of a six-inch second hand on a wall clock.

There are many ways to talk about how fast he is going:

RPM: How many revolutions per minute does he make? $\frac{1\text{ rev}}{1\text{ min}}$

How many revolutions per hour? $1\frac{\text{rev}}{\text{min}} \cdot \frac{60\text{ min}}{\text{hr}} = \frac{60\text{ rev}}{\text{hr}}$

ω What is his angular velocity in radians per hour? ω

$\frac{60\,\cancel{\text{rev}}}{\text{hr}} \cdot \frac{2\pi\text{ rad}}{1\,\cancel{\text{rev}}} = 120\pi\,\frac{\text{rad}}{\text{hr}}$

v What is his linear velocity in inches per hour?

$V = r\omega = 6\text{ in.} \frac{120\pi}{\text{hr}} = 720\pi\text{ in/hr}$

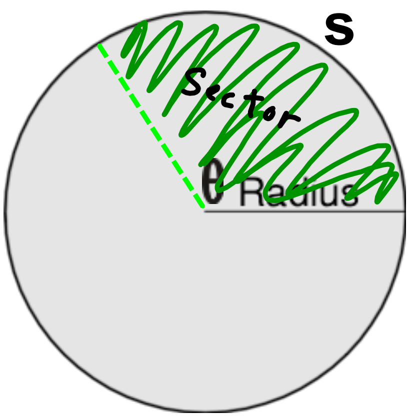
If he moves in two inches toward the center staying on the second hand, which of these will change? What will the new values be?

$r = 4''$

$\omega = 120\pi\text{ rad/hr}$

$v = 480\pi\text{ in/hr}$

A Sector of a Circle



The area inside the circle and inside the angle is called a sector of a circle. it is like a slice of pie.

The area of a sector is $A_{sector} = \frac{1}{2}r^2\theta$

Where r is the radius and the angle is in radians.

$$A = \frac{\theta \text{ degrees}}{360^\circ} \cdot \pi r^2$$

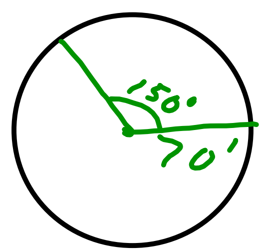
$$\frac{\theta}{\cancel{360^\circ}_{2}} \cdot \frac{\cancel{180^\circ}}{\cancel{\pi}} \cdot \cancel{\pi} r^2$$

$$= \frac{1}{2} r^2 \theta \quad \theta \text{ (radians)}$$

Let's think of a Rainbird sprinkler watering a large field.   The sprinkler takes 15 seconds to go back and forth. It is set for 150 degrees of coverage and the spray reaches out 70 feet.

$t = 15 \text{ sec}$
$\theta = 300°$
$r = 70'$

$\cancel{150}^{5}° \cdot \dfrac{\pi}{\cancel{180}_{6}°} = \dfrac{5\pi}{6}$



What is the angular velocity of the sprinkler?

$$\omega = \frac{\cancel{300}^{20}°}{\cancel{15} \text{ sec}} \cdot \frac{\pi}{\cancel{180}_{9}°} = \frac{\pi}{9} \text{ rad/sec}$$

What is the linear velocity of the bird flying back and forth at the end of the water stream?

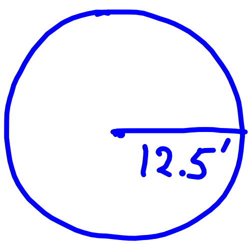
$$v = r\omega = 70'\left(\frac{\pi}{9}\right)/\text{sec} = \frac{70\pi}{9} \text{ ft/sec} \approx 24.4 \text{ ft/sec}$$

How much area will it water?

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(70)^2 \cdot \frac{5\pi}{6} \approx 6414 \text{ sq ft}$$

How fast are you going when sitting in a seat on a 25-foot Ferris Wheel which makes 5 rotations each minute?

diameter



$$\omega = \frac{5\ \cancel{\text{rotations}}}{1\ \text{min}} \cdot \frac{2\pi\ \text{rad}}{1\ \cancel{\text{rot}}} = \frac{10\pi}{\text{min}}$$

$$v = 12.5'\left(\frac{10\pi}{\text{min}}\right) = 125\pi\ \text{ft/min}$$

$$\approx 393\ \text{ft/min}$$