



Math 1050 ~ College Algebra

9 Real Zeros of Polynomials

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^m k = \frac{m(m+1)}{2}$$

$$\sum_{k=0}^n z^k = \frac{1 - z^{n+1}}{1 - z}$$

Learning Objectives

- Find possible (potential) rational zeros using the Rational Zeros Theorem.
- Find real zeros of a polynomial and their multiplicities.

We are now ready to determine the rational roots of a polynomial.

Rational Zeros Theorem

If $f(x)$ is a polynomial that has integer coefficients, every rational zero of $f(x)$ has the form $\frac{p}{q}$, where p is a factor of the constant term and q is a factor of the leading coefficient.

Ex 1: Use the Rational Zeros Theorem to determine the possible roots of these functions.

a) $f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$

b) $g(x) = 3x^3 + 3x^2 - 11x - 10$

This rule may further help you in eliminating some of the options when determining the roots of a polynomial.

Descartes Rule of Signs

Given a polynomial function with real coefficients and a constant term not zero:

- The number of positive real roots is equal to the number of variations in signs of $f(x)$ or less than that by an even number.
- The number of negative real roots is equal to the number of variations in signs of $f(-x)$ or less than that by an even number.

Ex 2: Determine how many positive and negative roots these functions are likely to have.

a) $f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$ b) $g(x) = 3x^3 + 3x^2 - 11x - 10$

Ex 3: Find all zeros for these functions.

a) $f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$ b) $g(x) = 3x^3 + 3x^2 - 11x - 10$

Multiplicity of Roots

A factor $(x-a)^k$, $k > 1$, yields repeated zero $x = a$ of multiplicity k .

Ex 4: Determine the roots and state the multiplicity of each. Write in factored form. $f(x) = x^5 - 8x^4 + 25x^3 - 38x^2 + 28x - 8$.