We are now ready to determine the rational roots of a polynomial.

**Rational Zeros Theorem**

If \( f(x) \) is a polynomial that has integer coefficients, every rational zero of \( f(x) \) has the form \( \frac{p}{q} \), where \( p \) is a factor of the constant term and \( q \) is a factor of the leading coefficient.

**Ex 1:** Use the Rational Zeros Theorem to determine the possible roots of these functions.

a) \( f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3 \)  
   b) \( g(x) = 3x^3 + 3x^2 - 11x - 10 \)
This rule may further help you in eliminating some of the options when determining the roots of a polynomial.

**Descartes Rule of Signs**

Given a polynomial function with real coefficients and a constant term not zero:
- The number of positive real roots is equal to the number of variations in signs of \( f(x) \) or less than that by an even number.
- The number of negative real roots is equal to the number of variations in signs of \( f(-x) \) or less than that by an even number.

Ex 2: Determine how many positive and negative roots these functions are likely to have.

\[ a) \quad f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3 \quad b) \quad g(x) = 3x^3 + 3x^2 - 11x - 10 \]

Ex 3: Find all zeros for these functions.

\[ a) \quad f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3 \quad b) \quad g(x) = 3x^3 + 3x^2 - 11x - 10 \]

**Multiplicity of Roots**

A factor \((x-a)^k\), \( k > 1 \), yields repeated zero \( x = a \) of multiplicity \( k \).

Ex 4: Determine the roots and state the multiplicity of each. Write in factored form. \( f(x) = x^5 - 8x^4 + 25x^3 - 38x^2 + 28x - 8 \).