Math 1050 ~ College Algebra

8 Using Synthetic Division to Factor Polynomials

Learning Objectives

- Use division to factor polynomials and determine zeros.
- Use synthetic division to simplify the division process.
- Use the Remainder Theorem to find function values of polynomials.
- Use the Factor Theorem to relate zeros to factors of polynomials.
When solving for the zeros of a function, it helps if we can break the function down into factors. Synthetic division will be useful to us.

### Factor Theorem
A polynomial $f(x)$ has a factor $(x-k)$ if and only if $f(k) = 0$.

### Remainder Theorem
If a polynomial $f(x)$ is divided by $(x-k)$, then the remainder $r = f(k)$.

**k ∈ ℝ (i.e. k is a constant)**

**Note:** Long division is ALWAYS useful for division of polynomials.

Synthetic division is only useful when dividing by $(x-k)$ where $k ∈ ℝ$.

Ex 1: Use long division to divide $(4x^3 + 10x^2 - 2x - 5)$ by $(2x^2 - 1)$.

\[
\begin{array}{c|cc cc}
2x^2 - 1 & 4x^3 & + 10x^2 & - 2x & - 5 \\
\hline & 2x^3 & + 0x^2 & - 2x & - 5 \\
\hline & -(2x^3 & + 0x^2 & - 2x) \\
\hline & 10x^2 & + 0x & - 5 \\
\hline & -(10x^2 & + 0x & - 5) \\
\hline & 0 & \\
\end{array}
\]

⇒ $(4x^3 + 10x^2 - 2x - 5) ÷ (2x^2 - 1) = 2x + 5$

Notice this process can help us factor the cubic (third degree) polynomial.

We get $(2x + 5)(2x - 1) = 4x^3 + 10x^2 - 2x - 5$
Ex 2: Divide \((x^3 + 4x^2 - 3x + 2)\) by \((x-3)\) in two ways.

**Long division**

\[
\begin{array}{r|rrrr}
  & x^2 & +7x & +18 \\
\hline
x-3) & x^3 & +4x^2 & -3x & +2 \\
       & x^3 & -3x^2 &   &   \\
\hline
       & 7x^2 & -3x & +2 \\
       & 7x^2 & -21x &   &   \\
\hline
       & 18x & +2 \\
       & 18x & -54 &   &   \\
\hline
       & 56 & & &   \\
\end{array}
\]

\[\Rightarrow (x^3 + 4x^2 - 3x + 2) \div (x-3) = x^2 + 7x + 18 + \frac{56}{x-3}\]

**Synthetic division**

\[
\begin{array}{r|rrrr}
  & 3 & 1 & 4 & -3 & 2 \\
\hline
  & 3 & 21 & 54 &   &   \\
\hline
  & 1 & 7 & 18 & 56 &   \\
\end{array}
\]

\[\Rightarrow (x^3 + 4x^2 - 3x + 2) \div (x-3) = x^2 + 7x + 18 + \frac{56}{x-3}\]
Ex 3: Use synthetic division to compute this quotient.

\[ (5x^3 + 6x + 8) \div (x + 2) \]

Write the result in the form \( f(x) = (x-k)(q(x)) + r(x) \)

\[ q(x) = \text{quotient} \]
\[ r(x) = \text{remainder} \]

Tips to help you:

1. Bring left # down
2. Multiply by root
3. In columns, add down

Ex 4: Use the remainder theorem and synthetic division to show that \((x+3)\) is a factor of this function.

\[ f(x) = 3x^3 + 5x^2 - 3x + 27 \]

\( \begin{array}{c|ccccc}
-3 & 3 & 5 & -3 & 27 \\
\hline
& -9 & 12 & -27 \\
\end{array} \)

\[ \Rightarrow 3x^3 + 5x^2 - 3x + 27 = (x+3)(3x^2 - 4x + 9) \]

\( \Rightarrow x+3 \) is a factor of \( f(x) \) since the remainder is 0.
Ex 5: Use division to show that $2/3$ is a solution of $48x^3 - 80x^2 + 41x - 6 = 0$.
Use the result to factor the polynomial completely and find all solutions.

\[
\begin{array}{c|cccc}
\frac{2}{3} & 48 & -80 & 41 & -6 \\
\hline
32 & \underline{48} & -32 & 6 \\
48 & -48 & 9 & \circ & \text{remainder} \\
\end{array}
\]

$\Rightarrow (x - \frac{2}{3})$ is factor of the cubic polynomial

\[
48x^3 - 80x^2 + 41x - 6 = 0
\]

\[
(x - \frac{2}{3})(48x^2 - 48x + 9) = 0
\]

\[
(x - \frac{2}{3})(3)(16x^2 - 16x + 3) = 0
\]

\[
3\left(x - \frac{2}{3}\right)(4x - 1)(4x - 3) = 0
\]

\[
(3x - 2)(4x - 1)(4x - 3) = 0
\]

$3x - 2 = 0 \quad \Rightarrow \quad 4x - 1 = 0 \quad \Rightarrow \quad 4x - 3 = 0$

$3x = 2 \quad \Rightarrow \quad x = \frac{2}{3}$

$4x = 1 \quad \Rightarrow \quad x = \frac{1}{4}$

$4x = 3 \quad \Rightarrow \quad x = \frac{3}{4}$