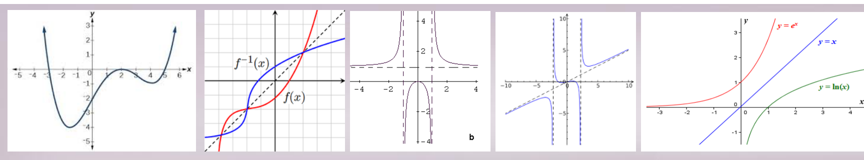


Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

7 Graphs of Polynomials

Learning Objectives

- Determine whether or not a function is a polynomial.
- Identify the degree, leading term, leading coefficient and constant term of a polynomial.
- Determine the existence of zeros using the Intermediate Value Theorem.
- Find the zeros and multiplicities of a polynomial; use multiplicity to determine the behavior of the graph at each zero.
- Identify the end behavior of a polynomial function.
- Sketch the graph of a polynomial function using zeros, multiplicities and end behavior.
- Solve applications that require finding the maximum or minimum value of a polynomial function.

A Polynomial Function and Vocabulary

$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0.$

Degree

Leading Term

Leading coefficient

Constant

Ex1: Determine which of these are polynomial functions and identify the degree, the leading term, the leading coefficient and the constant of those that are.

a) $f(x) = \sqrt{5}x^2 - 4x^3$ b) $f(x) = \sqrt[3]{x+2} + 1$ c) $f(x) = -3(x-2)^2 + 4x^6$

d) $f(x) = \dfrac{x-3}{x+2}$ e) $f(x) = \dfrac{6x^5 + 3x^2 - 1}{3}$ f) $f(x) = \pi$

Polynomial functions have the characteristic of being continuous and smooth.

The leading coefficient and the degree can tell us a lot about the graph of a polynomial, including its end behavior.

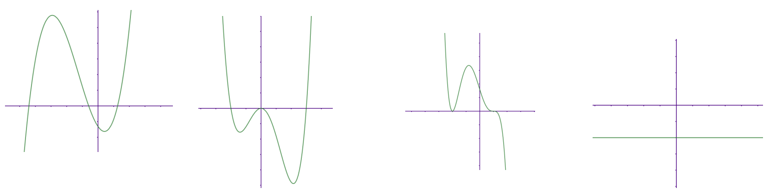
n is odd, $a > 0$

n is even, $a < 0$

n is even, $a > 0$

n is odd, $a < 0$

Ex 2: For each graph, guess at a likely degree, circle the x and y-intercepts, and identify the sign (+ or -) of the leading coefficient.



To graph a polynomial, it helps to determine the roots and the y-intercept.

Real Zeros of Polynomial Functions

Equivalent Statements: for $a \in \mathfrak{R}$, $f(x)$ a polynomial

- $x = a$ is a zero of $f(x)$.
- $x = a$ is a solution of $f(x) = 0$.
- $(x-a)$ is a factor of $f(x)$.
- $(a,0)$ is an x-intercept of the graph of $f(x)$.

Repeated Zeros

A factor $(x-a)^k$ for $k > 1$ yields a repeated zero, $x = a$ of multiplicity k.

- If k is odd, the graph crosses the x-axis at $x = a$.
- If k is even, the graph touches the x-axis at $x = a$.

Intermediate Value Theorem

Let $a, b \in \mathfrak{R}$ and $a < b$. If $f(x)$ is a polynomial and $f(x) \neq f(b)$, then over the interval $[a,b]$, f takes on every value between $f(a)$ and $f(b)$.

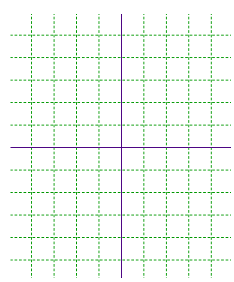
Ex3: For each function, describe the end-behavior, find all real zeros, including multiplicity, and the number of turning points on the graph.

a) $f(x) = (x+2)^2(x-1)^3$

b) $f(x) = -x(x+7)(x-3)^2$

Ex 4: Sketch the graph of f(x) by looking at the leading coefficients, finding the zeros, and perhaps plotting more points.

$f(x) = -48x^2 + 3x^4$



An Application Problem

Ex5: The profit (in millions of dollars) for a sport cap manufacturer can be modeled by $P(x) = -x^3 + 4x^2 + x$, where x is the number of caps they produce (in millions). They currently produce 4 million caps, making a profit of $4,000,000. What smaller number of caps could they make and still make the same profit?