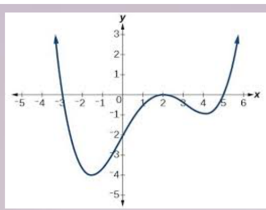
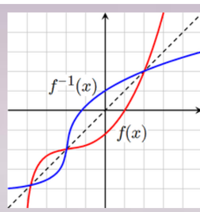
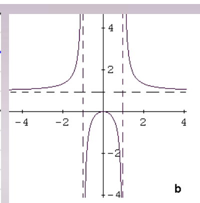
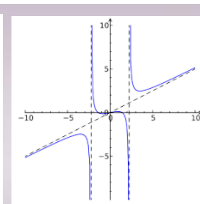
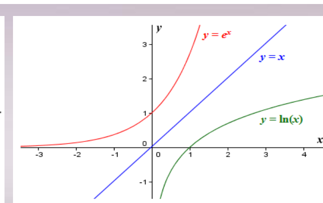










Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

7 Graphs of Polynomials

Learning Objectives

- Determine whether or not a function is a polynomial.
- Identify the degree, leading term, leading coefficient and constant term of a polynomial.
- Determine the existence of zeros using the Intermediate Value Theorem.
- Find the zeros and multiplicities of a polynomial; use multiplicity to determine the behavior of the graph at each zero.
- Identify the end behavior of a polynomial function.
- Sketch the graph of a polynomial function using zeros, multiplicities and end behavior.
- Solve applications that require finding the maximum or minimum value of a polynomial function.

A Polynomial Function and Vocabulary

$$\boxed{f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0.}$$

n must be a whole #

ex $f(x) = 3x^2 + 5x + 7$

$n = 2$
$a_2 = 3$
$a_1 = 5$ deg = 2
$a_0 = 7$
leading term $= 3x^2$
leading coeff. = 3
constant = 7

Degree

the highest exponent on x, in the function

Leading Term

the term w/ the highest exponent

Leading coefficient

the # multiplied by the highest power of x

Constant

the term with no x in it

Ex1: Determine which of these are polynomial functions and identify the degree, the leading term (lt), the leading coefficient (lc) and the constant of those that are.

a) $f(x) = \sqrt{5}x^2 - 4x^3$

$f(x) = -4x^3 + \sqrt{5}x^2$
is polynomial
deg = 3, lt. $= -4x^3$, lc $= -4$
const = 0

b) $f(x) = \sqrt[3]{x+2} + 1$

NOT a polynomial
(because x is inside a root)

c) $f(x) = -3(x-2)^2 + 4x^6$

$f(x) = 4x^6 - 3x^2 + 12x - 12$
this is polynomial
deg = 6, lt $= 4x^6$, lc = 4
const = −12

d) $f(x) = \dfrac{x-3}{x+2}$

This is NOT a polynomial because x is in denominator.

e) $f(x) = \dfrac{6x^5 + 3x^2 - 1}{3}$

$f(x) = 2x^5 + x^2 - \frac{1}{3}$
is polynomial
deg = 5, lt $= 2x^5$, lc = 2
const $= -1/3$

f) $f(x) = \pi$

(π is a constant)
is polynomial
deg = 0, lt $= \pi$, lc $= \pi$,
const $= \pi$

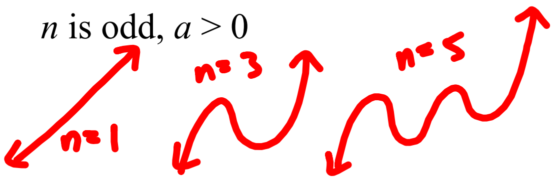
$-3(x-2)^2 = 3(x-2)(x-2)$
$= 3(x^2 - 2x - 2x + 4)$
$= 3(x^2 - 4x + 4)$
$= 3x^2 + 12x - 12$

degree = n , lc = a

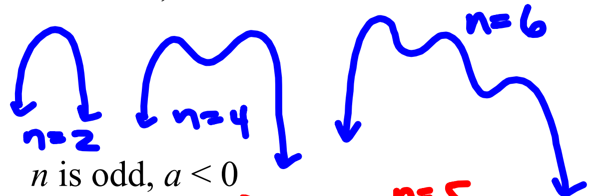
Polynomial functions have the characteristic of being continuous and smooth.

The leading coefficient and the degree can tell us a lot about the graph of a polynomial, including its end behavior.

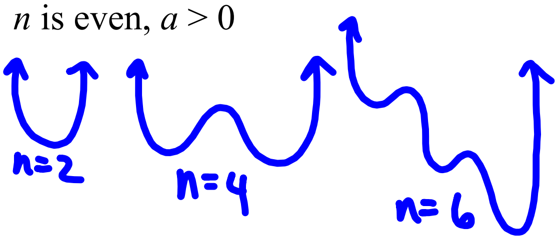
n is odd, $a > 0$



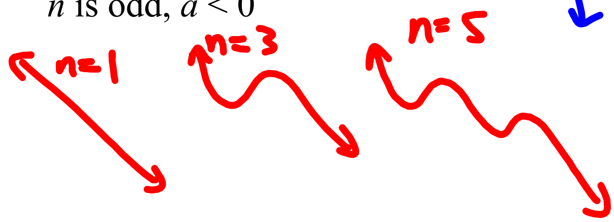
n is even, $a < 0$



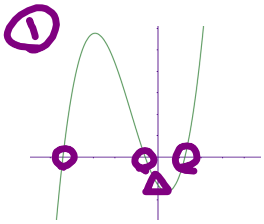
n is even, $a > 0$

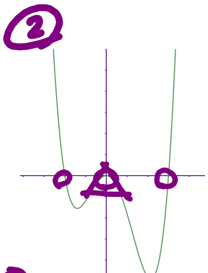


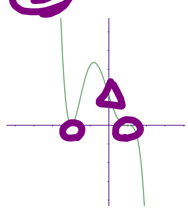
n is odd, $a < 0$

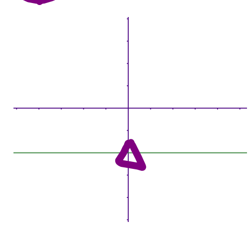


Ex 2: For each graph, guess at a likely degree, circle the x and y-intercepts, and identify the sign (+ or -) of the leading coefficient. Let a = leading coeff.

① 

② 

③ 

④ 

①	②	③	④
$n=3$, $a>0$ O x-intercepts (3 of them) Δ y-intercept (only 1 of them)	$n=4$, $a>0$ 3 x-intercepts 1 y-intercept	$n=5$, $a<0$ 2 x-intercepts 1 y-intercept	$n=0$ 0 x-intercepts 1 y-intercept

To graph a polynomial, it helps to determine the roots and the y-intercept.

Real Zeros of Polynomial Functions (roots = zeros; they're synonyms for polynomials)

Equivalent Statements: for $a \in \Re$, $f(x)$ a polynomial

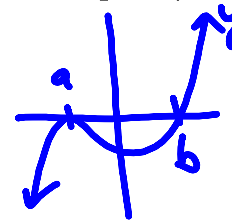
1. $x = a$ is a zero of $f(x)$.
2. $x = a$ is a solution of $f(x) = 0$.
3. $(x-a)$ is a factor of $f(x)$.
4. $(a,0)$ is an x-intercept of the graph of $f(x)$.

Repeated Zeros

A factor $(x-a)^k$ for $k > 1$ yields a repeated zero, $x = a$ of multiplicity k.

- If k is odd, the graph crosses the x-axis at $x = a$.
- If k is even, the graph touches the x-axis at $x = a$.

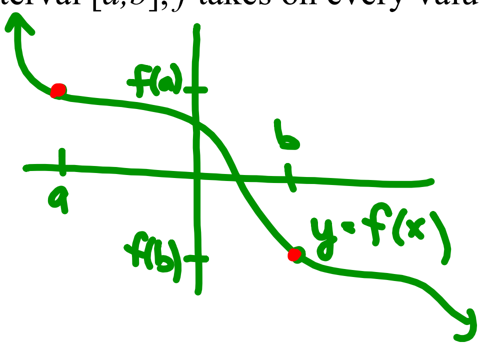
$(x-b)^m$ factor of $f(x)$ m odd



$(x-a)^k$ factor of $f(x)$ k is even

Intermediate Value Theorem

Let $a, b \in \Re$ and $a < b$. If $f(x)$ is a polynomial and $f(x) \neq f(b)$, then over the interval $[a,b]$, f takes on every value between $f(a)$ and $f(b)$.



in other words, we don't have a horizontal line as our polynomial.

Ex3: For each function, (a) describe the end-behavior, (b) find all real zeros, (c) including multiplicity, and the (d) number of turning points on the graph.

a) $f(x) = (x+2)^2(x-1)^3$

if we multiply this out, we'll get leading term of x^5.

$n=5$, $a>0$

(a) on left, down; on right, up.

(b) $(x+2)^2(x-1)^3=0$

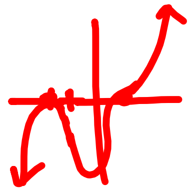
$x+2=0$ $x-1=0$

$x=-2, 1$

(c)

zero	multiplicity
-2	2 (exponent on x+2)
1	3 (exponent on x-1)

(d) 2 turning pts



b) $f(x) = -x(x+7)(x-3)^2$

if we multiply out, our leading term will be $-x(x)(x^2) = -x^4$

(a) $n=4$, $a<0$, on left & right sides, graph goes down forever

(b) $-x(x+7)(x-3)^2=0$

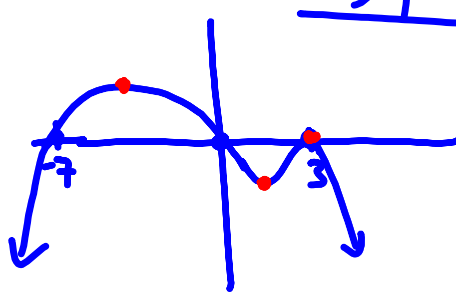
$-x=0$ $x+7=0$ $x-3=0$

$x=0,-7,3$ zeros of f

(c)

zeros	multiplicity	
0	1	(odd)
-7	1	(odd)
3	2	(even)

(d) 3 turning pts



Ex 4: Sketch the graph of f(x) by looking at the leading coefficient, finding the zeros, and perhaps plotting more points.

$f(x) = -48x^2 + 3x^4$

$n = 4, \quad a > 0$

$f(x) = 3x^4 - 48x^2$

$= 3x^2(x^2 - 16)$

* $f(x) = 3x^2(x-4)(x+4)$

roots: $3x^2(x-4)(x+4) = 0$

$x = 0, 4, -4$

multiplicity: 2, 1, 1

x-intercepts

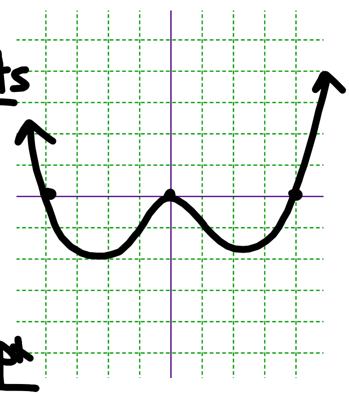
(0,0)

(4,0)

(-4,0)

y-intercept

(0,0)



$\Rightarrow$ graph crosses x-axis at $x=4$ and $x=-4$

graph only touches x-axis at $x=0$

An Application Problem

Ex5: The profit (in millions of dollars) for a sport cap manufacturer can be modeled by $P(x) = -x^3 + 4x^2 + x$, where x is the number of caps they produce (in millions). They currently produce 4 million caps, making a profit of \$4,000,000. What smaller number of caps could they make and still make the same profit?

i.e. if P=4, what are the x-values that will solve that eqn?

$4 = -x^3 + 4x^2 + x$

$0 = \underbrace{-x^3 + 4x^2}\underbrace{+x-4}$

$0 = -x^2(x-4) + (x-4)$

$0 = (x-4)(-x^2+1)$

$0 = (x-4)(1-x^2)$

$0 = (x-4)(1-x)(1+x)$

$x-4=0 \quad 1-x=0 \quad 1+x=0$

$x = 4, 1, \cancel{-1}$

can't make -1 caps

$\Rightarrow$ for same profit, the company can make 1,000,000 caps.