**6 Quadratic Functions**

**Math 1050 ~ College Algebra**

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<th>Learning Objectives</th>
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<td>- Graph a quadratic function through transformations of ( f(x) = x^2 ).</td>
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<td>- Change a quadratic function from general to standard form.</td>
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<td>- Find the vertex and axis of symmetry of a quadratic function.</td>
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<td>- Find the intercepts of a quadratic function.</td>
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<td>- Graph a quadratic function using vertex, axis of symmetry and intercepts.</td>
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<td>- Solve applications that require finding the maximum or minimum value of a quadratic function.</td>
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### Quadratic Functions

A polynomial function:  
\[
f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1x + a_0
\]

A quadratic function is a type of polynomial function where the degree = 2.  
\[
f(x) = ax^2 + bx + c \quad a,b,c \in \mathbb{R}, a \neq 0
\]

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<th>General Form</th>
<th>Standard Form</th>
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<td>( f(x) = ax^2 + bx + c )</td>
<td>( f(x) = a(x - h)^2 + k )</td>
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- **Axis of symmetry**
- **Vertex**
- **Concavity**

**Ex 1:** Determine the vertex, axis of symmetry and concavity of each of these.  
\[a) \ f(x) = 3x^2 + 6x - 4 \quad b) \ f(x) = -2(x+3)^2 - 4\]
Ex 2: Write the equation of this quadratic function in standard form, then use algebra to write it in general form.

Ex 3: Put this equation in standard form and sketch a graph of it.

\[ y = -2x^2 + 4x + 2 \]

Finding Roots of Quadratic Functions

To find the roots, solve for \( f(x) = 0 \).

If the expression on the left factors, set each factor equal to 0 and solve for \( x \).

If you prefer not to factor, or it does not factor, you can always use the Quadratic Formula.

\[ \text{Quadratic Formula: } \quad ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Ex 4: Determine the roots of each of these.

a) \( f(x) = 3x^2 + 5x - 4 \)  
b) \( f(x) = 9x^2 - 6x + 1 \)  
c) \( f(x) = 4x^2 - 6x - 3 \)
In the quadratic formula, the expression inside the radical is called the **discriminant**. It determines whether there is one real root, two real roots or no real roots.

Ex 5: Find the discriminant of the equations in example 4.

a) \( f(x) = 3x^2 + 5x - 4 \)  
b) \( f(x) = 9x^2 - 6x + 1 \)  
c) \( f(x) = 4x^2 - 6x - 3 \)

Ex 6: For this function, find the vertex, axis of symmetry, \( x \) and \( y \)-intercepts and sketch it.

\[
f(x) = -\frac{1}{2}(x^2 - 10x + 21)
\]

**An Application Problem**

Ex 7: The height of an object shot straight up in the air from a height of 128 feet at an initial velocity of 32 ft/sec is modeled by \( h(t) = -16t^2 + 32t + 128 \), where \( t = \) time.

Determine the maximum height the object reaches and the time it will hit the ground.