



Math 1050 ~ College Algebra

30 Binomial Expansion

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^m k = \frac{m(m+1)}{2}$$

$$\sum_{k=0}^n z^k = \frac{1-z^{n+1}}{1-z}$$

Learning Objectives

- Expand binomial powers using
 - Pascal's Triangle
 - Binomial Theorem
- Find an indicated term in the expansion of a binomial.

Binomial Expansion

The square and cube of a binomial are used so frequently, we may want to memorize them.

Ex 1: $(a+b)^2 =$ $(a+b)^3 =$

Given these, it is possible to determine the square or cube of more complicated binomials.

Ex 2: $(x-2y)^2 =$ $(3x+2y)^3 =$

Pascal's Triangle

$$(a+b)^0 =$$

$$(a+b)^1 =$$

$$(a+b)^2 =$$

$$(a+b)^3 =$$

$$(a+b)^4 =$$

Ex 3: Using the pattern above, expand these.

a) $(x-y)^5$

b) $(2x-3y)^4$

The Binomial Theorem

To get to this theorem, we need to introduce a new notation.

Factorial

The factorial of a non-negative integer is defined as:

$$0! = 1$$

$$n! = 1(2)(3)\dots(n)$$

Ex 4: Evaluate the factorials of the integers 1 through 6.

Ex 5: Evaluate

a) $\frac{5!}{3!}$

b) $(8-5)!$

c) $(8-3)!$

d) $\frac{17!}{14!}$

e) $\frac{100!}{96!4!}$

The Binomial Coefficient $\binom{n}{k}$ n, k are nonnegative integers
 $n \geq k$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Ex 6: Evaluate these binomial coefficients.

a) $\binom{5}{0}$ b) $\binom{10}{6}$ c) $\binom{8}{3}$ d) $\binom{8}{5}$ e) $\binom{8}{8}$

Ex 7: Show that $(a+b)^4 = \sum_{k=0}^4 \binom{4}{k} a^{4-k} b^k$

The Binomial Theorem $(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$

Ex 8: Use this theorem to expand $(3x-2)^5$.

Ex 9: Consider $(2x-y)^8$. Find the term that contains x^3 and the term that contains y^4 .