Math 1050 ~ College Algebra

30 Binomial Expansion

Learning Objectives

- Expand binomial powers using
  - Pascal’s Triangle
  - Binomial Theorem
- Find an indicated term in the expansion of a binomial.

![Graphs and equations]

**Binomial Expansion**

The square and cube of a binomial are used so frequently, we may want to memorize them.

**Ex 1:** \((a+b)^2 = \) \(a+b)^3 = \)

Given these, it is possible to determine the square or cube of more complicated binomials.

**Ex 2:** \((x-2y)^2 = \) \(3x+2y)^3 = \)
**Pascal's Triangle**

\[(a+b)^0 =\]
\[(a+b)^1 =\]
\[(a+b)^2 =\]
\[(a+b)^3 =\]
\[(a+b)^4 =\]

Ex 3: Using the pattern above, expand these.

a) \((x-y)^5\) \hspace{1cm} b) \((2x-3y)^4\)

---

**The Binomial Theorem**

To get to this theorem, we need to introduce a new notation.

**Factorial**

The factorial of a non-negative integer is defined as:

\[0! = 1\]
\[n! = 1(2)(3)...(n)\]

Ex 4: Evaluate the factorials of the integers 1 through 6.

---

Ex 5: Evaluate

a) \(\frac{5!}{3!}\) \hspace{1cm} b) \((8-5)!\) \hspace{1cm} c) \((8-3)!\) \hspace{1cm} d) \(\frac{17!}{14!}\) \hspace{1cm} e) \(\frac{100!}{96!4!}\)
The Binomial Coefficient \[
\binom{n}{k} = \frac{n!}{k!(n-k)!}
\] \(n, k\) are nonnegative integers \(n \geq k\)

Ex 6: Evaluate these binomial coefficients.

\begin{align*}
\text{a)} & \quad \binom{5}{0} \quad \text{b)} & \quad \binom{10}{6} \\
\text{c)} & \quad \binom{8}{3} \quad \text{d)} & \quad \binom{8}{5} \quad \text{e)} & \quad \binom{8}{8}
\end{align*}

Ex 7: Show that \((a + b)^4 = \sum_{k=0}^{4} \binom{4}{k} a^{4-k} b^k\)

The Binomial Theorem \((a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k\)

Ex 8: Use this theorem to expand \((3x-2)^5\).

Ex 9: Consider \((2x-y)^8\). Find the term that contains \(x^3\) and the term that contains \(y^4\).