Math 1050 ~ College Algebra

29 Series

Learning Objectives

• Use summation notation.
• Find the sum of a finite arithmetic sequence.
• Solve applications of arithmetic series.
• Find the value of an infinite geometric series with a finite sum.
• Find the sum of a finite geometric sequence.
• Solve applications of geometric series.

Summation Notation

\[ \sum_{n=1}^{p} a_n = a_1 + a_2 + a_3 + \cdots + a_p \]

\[ \sum_{n=1}^{p} a_n = a_1 + a_2 + a_3 + \cdots + a_{p-1} + a_p \]

\[ \sum_{n=1}^{p} a_n = a_1 + a_2 + a_3 + \cdots \]

Ex 1: Find the following sums.

\[ a) \quad \sum_{n=2}^{6} (2n - 1) \quad b) \quad \sum_{k=1}^{4} (-1)^k (2k) \quad c) \quad \sum_{k=0}^{5} 2^k \]

Ex 2: Write the following sums using summation notation. Assume the terms in each result from an arithmetic or geometric sequence.

\[ a) \quad 9 - 6 + 4 - \frac{8}{3} + \frac{16}{9} \quad b) \quad \frac{19}{2} + \frac{11}{2} + \frac{3}{2} - \frac{5}{2} + \cdots - \frac{29}{2} \quad c) \quad \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots \]
Properties of Summation

\[ \sum_{n=j}^{p}(a_n \pm b_n) = \sum_{n=j}^{p} a_n \pm \sum_{n=j}^{p} b_n \]

\[ \sum_{n=j}^{p} a_n = \sum_{n=j}^{p} a_n + \sum_{n=j}^{p} a_n \text{, for any integer } j \leq h < p \]

\[ \sum_{n=j}^{p} c a_n = c \sum_{n=j}^{p} a_n \quad \text{c is a constant} \]

\[ \sum_{n=j}^{p} a_n = \sum_{n=j+h}^{p+h} a_{n-h} \quad \text{for any integer } h \text{ (if } p = \infty, \text{ replace } p+h \text{ with } \infty) \]

Ex 3: Use the properties above to state these in another way.

a) \[ \sum_{k=1}^{10} \frac{k^2}{3} \]

b) \[ \sum_{k=1}^{10} \left(2k - \frac{1}{k^2}\right) \]

c) \[ \sum_{j=2}^{10} (j+1) + \sum_{j=2}^{10} \frac{2}{j^2} \]

Arithmetic Series

Ex 4: Add the first hundred integers.
**Sum of a Finite Arithmetic Sequence**

\[ S_n = \sum_{j=1}^{n} a_j = \frac{n}{2} (a_1 + a_n) = \frac{n}{2} (2a_1 + (n-1)d), \quad n \geq 2 \quad \text{where } a_j = a_1 + (j-1)d \]

**Ex 5:** Find the \( n^{th} \) partial sum for each of these.

a) \( \sum_{n=2}^{10} (2n - 1) \)  

b) \( \frac{19}{2} + \frac{11}{2} + \frac{3}{2} + \frac{5}{2} + \cdots, n = 10 \)

c) The sequence \( \{a_n\} \) where \( a_1 = 15, a_{10} = 312, n = 50 \)

**Sum of a Finite Geometric Sequence**

\[ S_n = \sum_{j=1}^{n} a_j = a \frac{1-r^n}{1-r} \quad \text{where } a_j = a \cdot (r^{j-1}) \]

**Sum of an Infinite Geometric Sequence**

\[ S = \sum_{j=1}^{\infty} a_j = a \frac{1}{1-r}, \quad -1 < r < 1 \quad \text{where } a_j = a \cdot (r^{j-1}) \]

**Ex 6:** Determine each sum.

a) \( \sum_{n=0}^{\infty} \left( \frac{1}{10} \right)^n \)  

b) \( \sum_{n=0}^{\infty} \left( \frac{2}{3} \right)^n \)

c) 1.38  
   **Hint:** 1.38 = 1.3 + 0.08 + 0.008 + 0.0008 + ...

d) \( \sum_{k=0}^{\infty} 3^k \)
Applications of Series

Ex 7: You are trying to break a bad habit. Two relatives offer to help with a financial incentive, but you must choose only one. How much is each offer? Which would you take?

a) Your Great Auntie Mare offers to give you $1.00 on the first day of February and each day thereafter, she will give you one dollar more than she did the day before.

b) Your Uncle Ulysses offers to give you 1 cent on the first day of February and each day thereafter, he will give you double what he gave you the day before.