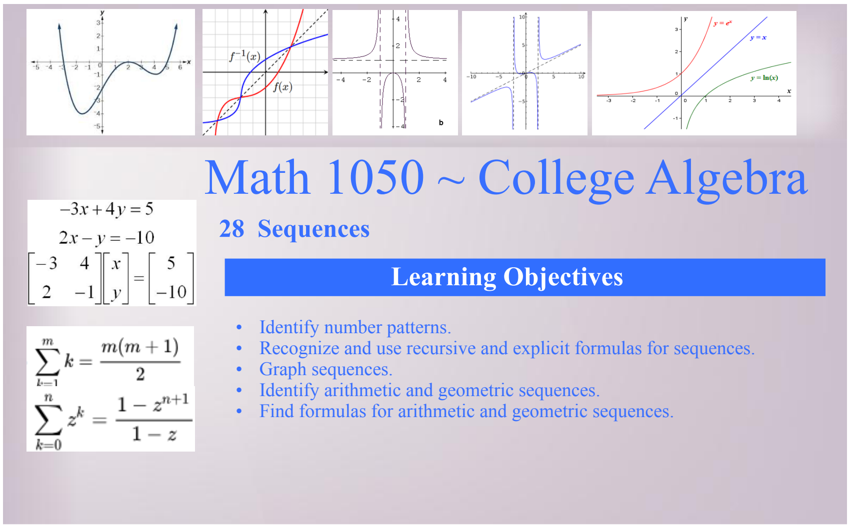


Number Patterns

An ordered collection of numbers or events is called a sequence. There are many interesting numeric sequences. If it goes on indefinitely, it is called an infinite sequence.

Ex 1: For each sequence, determine how to find the next term and state two more terms.

a) 2, 4, 8, ...

b) $\frac{2}{9}, \frac{3}{8}, \frac{4}{7}, \frac{5}{6}, \ldots$

c) 2, 4, 16, ...

d) 19, 11, 3, -5, -13, ...

e) 16, -8, 4, -2, ...

f) 1, 1, 2, 3, 5, 8, ...

g) 27, 18, 12, 8, ...

A recursive formula defines each new term by one or more of the previous terms.

1, 1, 2, 3, 5, 8, ...

An explicit formula describes how to find any term of the sequence.

2, 4, 8, ...

Ex 2: Write five terms for each sequence. Identify as recursive or explicit.

a) $a_n = (-1)^n \left(\dfrac{n}{n+1} \right)^n$

b) $a_n = \dfrac{1}{a_{n-1}}$

Ex 3: Write an explicit and a recursive formula for this sequence.

2, -4, 6, -8, ...

A sequence is a function with the domain of all natural numbers or a consecutive subset of the natural numbers.

Here are different ways to describe odd natural numbers as a sequence. Note that the fourth option allows us to have a finite sequence or one beginning with 0.

$f(n) = 2n - 1,\ n = 1, 2, 3, \ldots$

$a_n = 2n - 1,\ n = 1, 2, 3, \ldots$

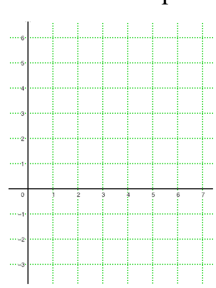
$\{2n-1\}_{n=1}^{\infty}$

$\{2n-1\}$

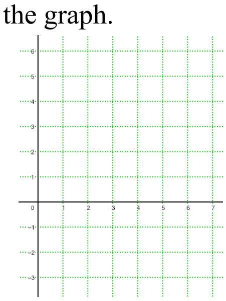
Since a sequence is a function, we can graph it. Note that the graph will be points, not connected with a curve.

Ex 4: Write five terms of each sequence and plot each on the graph.

a) $\{(-1)^n(5-2n)\}_{n=1}^{\infty}$



b) $\left\{ \dfrac{3n}{n+1} \right\}_{n=1}^{\infty}$



Arithmetic Sequence

$\{a_n\}$ is an arithmetic sequence if successive terms have the same difference.

$a_n = a_{n\text{-}1} + d$ with a_1 given

Ex 4: Which of these sequences are arithmetic? For those that are, find d and a_{20}.

a) 5.3, 5.7, 6.1, 6.5, 6.9, ...

b) ln 2, ln 5, ln 8, ln 11, ...

c) 800, 400, 200, 100, 50, ...

d) $\frac{19}{2}, \frac{11}{2}, \frac{3}{2}, -\frac{5}{2}, -\frac{13}{2}, \ldots$

Ex 5: Find an explicit formula for a_n, such that $\{a_n\}$ is arithmetic and $a_1 = 0$, $d = -2/3$. Write the first five terms.

Ex 6: Find an explicit formula for the arithmetic sequence such that $a_2 = 3$ and $a_7 = 33$.

Geometric Sequence

$\{a_n\}$ is a geometric sequence if successive terms have the same quotient (ratio).

$a_n = a_{n\text{-}1}r$ with a_1 given

Ex 7: Which of these are geometric? If they are, determine r and find a_{10}.

a) $\frac{1}{5}, \frac{2}{7}, \frac{3}{9}, \frac{4}{11}, \ldots$

b) $\frac{19}{2}, \frac{11}{2}, \frac{3}{2}, -\frac{5}{2}, -\frac{13}{2}, \ldots$

c) 800, 400, 200, 100, 50, ...

d) $9, -6, 4, -\frac{8}{3}, \ldots$

Ex 8: Write the first six terms of the geometric sequence with $a_1 = 6$, $r = -\frac{1}{4}$

Ex 9: If $a_2 = 3$ and $a_5 = \frac{3}{64}$, and $\{a_n\}$ is geometric, find a_1, a_7 and a formula for a_n.

Ex 10: Identify each sequence from example 1 as geometric, arithmetic or neither and state a reason.

a) 2, 4, 8, ...

b) $\frac{2}{9}, \frac{3}{8}, \frac{4}{7}, \frac{5}{6}, \ldots$

c) 2, 4, 16, ...

d) 19, 11, 3, -5, -13, ...

e) 16, -8, 4, -2, ...

f) 1, 1, 2, 3, 5, 8, ...

g) 27, 18, 12, 8, ...