Math 1050 ~ College Algebra

27 Partial Fractions

Learning Objectives

- Decompose a rational expression with denominator of non-repeated linear factors into a sum of partial fractions.
- Decompose a rational expression with denominator of repeated linear factors into a sum of partial fractions.
- Decompose a rational expression with denominator of non-repeated irreducible quadratic factors into a sum of partial fractions.
- Decompose a rational expression with denominator of repeated irreducible quadratic factors into a sum of partial fractions.

Partial Fraction Decomposition

Distinct Linear Factors

There are times, in future math classes, when you would like to break a rational expression into a sum of simpler fractions. We will begin with a proper fraction, where the degree of the numerator is less than the degree of the denominator. The first step is to factor the denominator and write it as a sum of $n$ terms for an $n^{th}$ degree denominator.

$$
\frac{p(x)}{q(x)} = \frac{A}{a_1x + b_1} + \frac{B}{a_2x + b_2} + \frac{C}{a_3x + b_3} + \cdots
$$

Ex 1: Determine $A$ and $B$ for this proper fraction. $\frac{3x - 1}{x(x - 4)} = \frac{A}{x} + \frac{B}{x - 4}$

If the fraction is improper, we must do long division first.

Ex 2: Write the partial fraction decomposition for this expression. $\frac{x^2 + 1}{x^3 - x}$
Repeated Linear Factors

\[
\frac{p(x)}{q(x)} = \frac{A}{ax + b} + \frac{B}{(ax + b)^2} + \frac{C}{(ax + b)^3} + \cdots + \frac{N}{(ax + b)^r}
\]

Ex 3: Resolve into partial fractions \( \frac{2x^2 + 7x + 4}{(x + 1)^3} \).

Unique Irreducible Quadratic Factors

\[
\frac{p(x)}{q(x)} = \frac{Ax + B}{a_1x^2 + bx + c_1} + \frac{Cx + D}{a_2x^2 + b_2x + c_2} + \cdots
\]

Ex 4: Write the partial fraction decomposition of \( \frac{-x^4 + 4x^3 - 2x + 6}{x^2(x^2 + 2)} \).
Repeated Irreducible Quadratic Factors

\[ \frac{p(x)}{q(x)} = \frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \frac{A_3x + B_3}{(ax^2 + bx + c)^3} + \cdots + \frac{A_nx + B_n}{(ax^2 + bx + c)^n} \]

Ex 4: Write the partial fraction decomposition of \( \frac{x^2 + x + 2}{(x^2 + 2)^2} \).