Math 1050 ~ College Algebra

23 Systems of Linear Equations: Augmented Matrices

Learning Objectives

- Write a system of linear equations as an augmented matrix.
- Perform row operations on a matrix.
- Convert an augmented matrix to row echelon form.
- Convert an augmented matrix to reduced row echelon form.
- Use matrix row operations to solve systems of linear equations.

Solving three equations in three variables takes a bit of writing. We can reduce some of that by writing the set of equations as an augmented matrix. A matrix is a structured array of numbers grouped by square brackets. We only write the coefficients and constants.

Ex 1a: Write this set of equations as an augmented matrix.

\[
\begin{align*}
2x - y + z &= 4 \\
x + 3y - 2z &= -3 \\
3x + 2y + 2z &= 6 \\
\end{align*}
\]

We can operate on this matrix using the same set of rules that we used for Gaussian elimination in the last section.

1. Exchange two rows.
2. Multiply a row by a nonzero constant.
3. Temporarily multiply a row by a nonzero constant and add it to another row, replacing either of those rows with the result.

Ex 1b: Solve this system of equations from 1a by using these options.
Row Echelon Form

Ex 2: Solve for $x, y$ and $z$ in these augmented matrices representing a set of equations.

a) $\begin{bmatrix} 1 & 0 & 0 & : & -3 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 1 & : & 0 \end{bmatrix}$  

b) $\begin{bmatrix} 1 & -2 & 3 & : & 9 \\ 0 & 1 & 3 & : & 5 \\ 0 & 0 & 1 & : & 2 \end{bmatrix}$

These are examples of row echelon form and reduced row echelon form. Our goal is to reduce a set of equations to one of these forms, using Gaussian elimination.

Ex 3: Solve this set of equations by reducing them to each of the forms above.

\[
\begin{align*}
2x - 2y + z &= -9 \\
x + y + 2z &= -5 \\
x - z &= 11
\end{align*}
\]

Ex 4: Set up this set of equations as an augmented matrix and put in reduced row echelon form.

\[
\begin{align*}
A &= 2 \\
B &= -1 \\
3A + 2C &= 0 \\
2B - D &= -5
\end{align*}
\]
What happens if the system has infinite solutions or no solutions?

Ex 5: Write the solution to each system below.

\[
\begin{align*}
a) \begin{bmatrix} 1 & 0 & 0 & : & -3 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 1 & : & 0 \end{bmatrix} & \quad b) \begin{bmatrix} 1 & 0 & 0 & : & -3 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 0 & : & 2 \end{bmatrix} & \quad c) \begin{bmatrix} 1 & 0 & 0 & : & -3 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}
\end{align*}
\]