Math 1050 ~ College Algebra

22 Systems of Linear Equations and Applications

Learning Objectives

- Solve systems of three linear equations in three variables.
- Interpret solutions to 3×3 systems of linear equations.
- Solve applications of linear equations in three variables.
A linear equation in three variables, \( x, y \) and \( z \), is an equation of the form 
\( ax + by + cz = d \) where \( a, b, c \) and \( d \) are constants and at least one of \( a, b \) and \( c \) is nonzero. Such an equation represents a plane in 3-D space.

Here are some possibilities of the intersection of three planes.

We will solve these equations by using linear combinations. Your goal is to solve for \( x, y \) and \( z \). This procedure is called **Elimination**.

Here are the legitimate actions you may take.

1. Exchange two rows.
2. Multiply a row by a nonzero constant.
3. Temporarily multiply a row by a nonzero constant and add it to another row, replacing either of those rows with the result.

Ex 1: Solve this system by using Elimination.

\[
\begin{align*}
\text{(1)} & \quad x - y + z = 4 \\
\text{(2)} & \quad x + 3y - 2z = -3 \\
\text{(3)} & \quad 3x + 2y + 2z = 6 \\
\end{align*}
\]

Side:
\[
\begin{align*}
\text{(4)} & \quad x + 3y - 2z = -3 \\
\text{(5)} & \quad 3x + 2y + 2z = 6 \\
\text{(6)} & \quad 4x + 5y = 3 \\
\end{align*}
\]

\[
\begin{align*}
\text{(7)} & \quad 2x - 2y + 2z = 8 \\
\text{(8)} & \quad x - y + z = 4 \\
\text{(9)} & \quad x + 3y - 2z = -3 \\
\text{(10)} & \quad 4x + 5y = 3 \\
\end{align*}
\]

Solution: \( (2, -1, 1) \)
Ex 2: Solve \[\begin{align*}
3x+6y-3z &= -12 \\
x-2y+z &= 4
\end{align*}\]

Ex 3: Solve \[\begin{align*}
x-2y-z &= -5 \\
2x+y+z &= 5
\end{align*}\]

Because \(D=-5\) (i.e. \(D=-5\) is not a true statement), we know there is no point of intersection. \(\text{N.S.}\)
Ex 4: Find the equation of the parabola, \( y = ax^2 + bx + c \) that passes through these three points, \((0,3), (1,4)\) and \((2,3)\).

\[
\begin{align*}
\text{①} \quad 3 &= 0 + 0 + c \quad 3 = c \\
\text{②} \quad 4 &= a(1) + b(1) + c \quad 4 = a + b + c \\
\text{③} \quad 3 &= a(4) + b(2) + c \quad 3 = 4a + 2b + c \\
\end{align*}
\]

choose substitution method.

\[
\begin{align*}
\text{②} \quad 4 &= a + b + 3 \quad 1 = a + b \\
\Rightarrow \quad d &= 4a + 2b + 3 \\
\text{③} \quad 2b &= -4a \\
\Rightarrow \quad b &= -2a \\
\text{①} \quad 1 &= a + (2a) \quad 1 = -a \\
\Rightarrow \quad 4 &= -1 \\
\Rightarrow \quad b &= -2(-1) \\
\Rightarrow \quad b &= 2 \\
\end{align*}
\]

\( \Rightarrow \) we have parabola

\[
y = -x^2 + 2x + 3
\]