Math 1050 ~ College Algebra

1 Introduction to Functions

Learning Objectives

- Determine whether a relation represents a function.
- Use the vertical line test to identify graphs of functions.
- Find the domain and range from the graph of a function.
- Find input and output values of a function.
- Find the domain from the equation of a function.
A **relation** is a set of ordered pairs. The set of first components of the ordered pairs is called the **domain** and the set of second components of the ordered pairs is called the **range**.

- Input value $\|\$ independent variable
- Output value $\|\$ dependent variable

(\text{depends on input})

**Ex1**: For each of these, state whether it is a relation, and if it is, list the elements in the domain and in the range.

a) $\{(1,5), (5,-2), (5,4), (3,2)\}$

- **Yes, it's a relation.**
- $D:\{1,5,3\}$  $R:\{5,-2,4,2\}$

b) Bud 15  
   May 16  
   Ezi 17  
   Zhu 18  
   Tia 19  

d) Input values: days of the week  
   Output values: final letter in word

- **D**: \{Sunday, Monday, Tuesday, Wed., Thurs., Fri., Sat.\}
- **R**: \{y\}

e) \{name, rank, serial number\}

**not a relation**
A **function** is a relation in which any two ordered pairs with the same first component also have the same second component.

A function has only one output for any given input (fn means function)

Ex 2: From example 1, which of the relations are functions?

a) \{(1,5), (5,-2), (5,4), (3,2)\}

*not* a fn, because input 5 has 2 outputs

b) Bud 15
May 16
Ezi 17
Zhu 18
Tia 19

yes, is a fn because every input has only one output

c) ![Graph](image)

Yes, it’s a fn because every input has only one output

d) Input values: days of the week
Output values: final letter in word

ex (Tuesday, y) (Wed, y)

Yes, a fn.
An equation in two variables can be a relation as can a 2-dimensional graph.

Ex 3: Which of these are functions?

a) \( x + 3 = y^2 \)
   \[ y = \pm \sqrt{x+3} \]
   \( \Rightarrow \) there are 2 outputs for most inputs \( \Rightarrow \) it is not a fn.

b) \( 2y = \sqrt{x-1} \)
   \[ y = \frac{1}{2} \sqrt{x-1} \]
   \( \Rightarrow \) for every \( x \)-value, we get back one \( y \)-value \( \Rightarrow \) is a fn

c) \( x^2 + y^2 = 9 \)
   \( \Rightarrow \) if \( x = \sqrt{5} \), then \( x^2 = 5 \)
   \( 5 + y^2 = 9 \) \( \iff \) \( y^2 = 4 \) \( \iff \) \( y = \pm 2 \)

So one particular \( x \)-value yielded two \( y \)-values \( \Rightarrow \) is not a fn

d) \( \{(3,1), (2,1), (5,1), (6,2)\} \)
   every input has only one output \( \Rightarrow \) this is a fn
**The Vertical Line Test:** A graph represents a function if no vertical line intersects it at more than one point.

Ex 4: Use the vertical line test to determine if these relations are functions.

\[ R_1 = \{(1,5), (5,-2), (5,4), (3,2)\} \quad R_2 = \{(3,1), (2,1), (5,1), (-3,2)\} \]
Function Notation

\[ f \text{ is a fn that takes an input } (x) \text{ and maps it to an output } (y). \]
\[ y = f(x) \quad \text{(read } f \text{ of } x\text{)} \]

Ex 5: Evaluate these functions for the given values.

a) \( f(x) = \sqrt{x+8} + 2 \)

\[ f(-8) = \sqrt{-8+8} + 2 = 0 + 2 = 2 \]
\[ f(x-8) = \sqrt{(x-8)+8} + 2 = \sqrt{x} + 2 \]
\[ f(a) = \sqrt{a+8} + 2 \]

b) \( g(2) = -3 \)

\( g(0) = \) undefined

\( g(a) = -2 \) for \( a = 3 \)

Output = -2, input?
Domain of Functions

The domain of a function is the set of all input values for which the function is defined.

**Implicit domain**  
(domain that's implied by computations needed in the function)

**Explicit domain**  
handed to you

Ex 6: Determine the domain for each of these functions and identify as implicit or explicit.

a) \( f(x) = \sqrt[3]{x + 4} \)
   
   D: \( x \in \mathbb{R} \)  
   
   note: we can take cube root of any number

b) \( p(x) \)
   
   D: \( x > 1 \)  
   
   (or \( x \in (1, \infty) \))

(c) \( q(x) = \frac{3}{x^2 - 2x} \)
   
   D: \( x \in \mathbb{R} \), \( x \neq 0 \), \( x \neq 2 \)  
   
   (because those \( x \)-values make denominator \( 0 \))

   \( (-\infty, 0) \cup (0, 2) \cup (2, \infty) \)

   (implicit)

(d) \( f(x) = \frac{\sqrt{x+4}}{4+x} \)
   
   \( x+4 \geq 0 \)  
   
   \( x \geq -4 \)

   D: \( x > -4 \)
   
   (or \( x \in (-4, \infty) \))

(e) \( h(x) = 5x - 3 \), \( x > -1 \)
   
   D: \( x > -1 \)  
   
   (or \( x \in (-1, \infty) \))

(Explicit)