

Math 1050 ~ College Algebra

19 Logarithmic Equations and Functions

Learning Objectives

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^m k = \frac{m(m+1)}{2}$$

$$\sum_{k=0}^n z^k = \frac{1 - z^{n+1}}{1 - z}$$

- Determine the domain of a logarithmic function.
- Determine x- and y-intercepts of logarithmic functions.
- Graph logarithmic functions.
- Solve logarithmic equations.
- Solve applications of logarithmic functions.

Determine the Domain of a Logarithmic Function

Remember $\log x$ is only defined for $x > 0$.

$\log \heartsuit$ domain
 $\heartsuit > 0$

Ex 1: Determine the domain of each of these.

a) $f(x) = \log_5(x+3)$

domain: $x+3 > 0$
 $x > -3$
or $(-3, \infty)$

b) $g(x) = \log(x^2+3)$

domain:
 $x^2+3 > 0$
already true
 $x \in \mathbb{R}$ or $(-\infty, \infty)$

c) $h(x) = \ln x - \ln(x+1)$

domain:
① $x > 0$ and
② $x+1 > 0$
 $x > -1$
 $\rightarrow x > 0$
or $(0, \infty)$

Solving Logarithmic Equations

Ex 2: We will solve this equation by going through the basic steps.

$\log(5x) + \log(x-1) = 2$

a. Use properties of logs to condense logs into a single log expression on one side of the equation.

$\log(5x(x-1)) = 2$

b. Exponentiate both sides with the base matching the base of the log.

OR $10^{\log(5x^2-5x)} = 10^2 \Leftrightarrow 5x^2-5x = 100$

c. Rewrite as an equivalent exponential equation. (rewrite (a)) using defn of log

$10^2 = 5x^2 - 5x$

d. Solve and check.

$5x^2 - 5x - 100 = 0$
 $5(x^2 - x - 20) = 0$
 $5(x-5)(x+4) = 0$

MUST DO:
check answers w/ domain of original eqn.

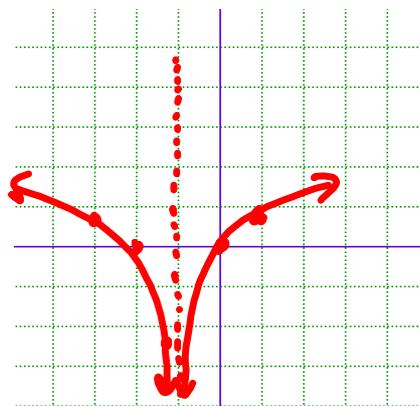
$x-5=0$ or $x+4=0$
 $x=5, \cancel{x}$

$\log(5x) + \log(x-1) = 2$
 $x=5: 5(5) > 0 \checkmark$
 $5-1 > 0 \checkmark$
 $x=-4: 5(-4) < 0 \Rightarrow$ throw away
 $x=-4$

$x=5$

Ex 3: Graph this logarithmic function by following these steps. $f(x) = \ln |x + 1|$

- Determine the domain.
- Find the x - and y - intercepts.
- Determine any asymptotes.
- Plot a few points and sketch the curve.



(a) $|x+1| > 0 \Rightarrow x \neq -1$

$(-\infty, -1) \cup (-1, \infty)$

(b) y -int: $(0, 0)$

$y = \ln|0+1| = \ln 1 = 0$

x -int: $(0, 0)$
 $(-2, 0)$

$0 = \ln|x+1|$

$e^0 = |x+1|$

$1 = |x+1|$

$x+1=1$ $-(x+1)=1$

$x=0$ $-x-1=1$

$-x=2$

$x=-2$

$y = \ln|x+1|$

(c) VA: $x = -1$

(d)

x	y
-3	$\ln 2 = \ln -2 $
1	$\ln 2 \approx 0.69$
-1.1	$\ln 0.1 \approx -2.3$

Ex 4: Solve for x.

a) $\log_3 x - \log_3(x+1) = 2$

$$\log_3 \left(\frac{x}{x+1} \right) = 2$$

$$3^2 = \frac{x}{x+1}$$

$$9(x+1) = x$$

$$9x+9 = x$$

$$8x+9 = 0$$

$$8x = -9$$

$$x = -\frac{9}{8}$$

doesn't satisfy domain

check answer:

original eqn $\log_3 x - \log_3(x+1) = 2$

notice: ~~$\log_3 \left(\frac{-9}{8} \right)$~~

\Rightarrow answer is **N.S.**

b) $\ln(x+4) - \ln(x-2) = \ln x$

$$\ln \left(\frac{x+4}{x-2} \right) = \ln x$$

(use 1 to 1 property)

$$\cancel{(x-2)} \frac{x+4}{\cancel{x-2}} = x(x-2)$$

$$x+4 = x^2 - 2x$$

$$0 = x^2 - 3x - 4$$

$$0 = (x-4)(x+1)$$

$$x-4=0 \text{ or } x+1=0$$

$$x=4, -1$$

check answers:

eqn: $\ln(x+4) - \ln(x-2) = \ln x$

$x=4$: $\ln 8 - \ln 2 = \ln 4$ ✓

$x=-1$: ~~$\ln 3 - \ln(-3) = \ln(-1)$~~

does not work

\Rightarrow answer is **$x=4$**

Ex 5: Marilyn is saving for her retirement by depositing \$500 per month into an account earning 5.4% annual interest compounded monthly. The time it takes for such an account to grow to be worth S dollars is given by this equation.

$$t = \frac{1}{12} \log_{\left(1 + \frac{r}{12}\right)} \left[\frac{Sr}{12P} + 1 \right]$$

r = annual interest rate

t = time (in years)

P = monthly payments

$$r = 0.054$$

$$P = \$500$$

$$S = 2,000,000$$

How many years will she need to keep making these monthly deposits for the account to be worth two million dollars?

$$t = ?$$

$$t = \frac{1}{12} \log_{1.0045} \left[\frac{2000000(0.054)}{12(500)} + 1 \right]$$

$$t = \frac{1}{12} \left[\frac{\ln 19}{\ln 1.0045} \right]$$

$$t \approx 34.7 \text{ years}$$

$$1 + \frac{r}{12} = 1 + \frac{0.054}{12} = 1.0045$$

$$\frac{2000000(0.054)}{12(500)} + 1 = 19$$