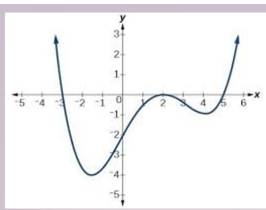
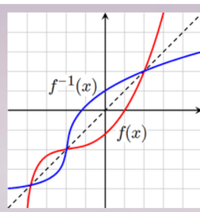
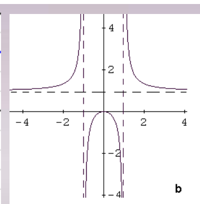
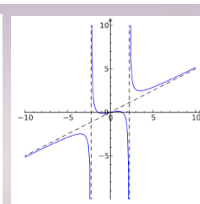
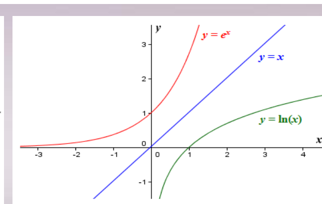










Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

18 Exponential Equations and Functions

Learning Objectives

- Solve exponential equations.
- Determine x- and y-intercepts of exponential functions.
- Graph exponential functions.
- Solve applications of exponential functions.

Strategy for Solving Exponential Equations

1. If you can get an exponential equation in the form of $b^n = b^m$, then you may use the one-to-one property, and $n = m$.

$\frac{1}{4} = \frac{1}{2^2} = 2^{-2}$

Ex 1: Solve for x.

a) $3^{2x} = 27$

$3^{2x} = 3^3$

$2x = 3$

$\boxed{x = \frac{3}{2}}$

b) $5^{x-1} = 125^{x+1}$

$5^{x-1} = (5^3)^{x+1}$

$5^{x-1} = 5^{3x+3}$

$x-1 = 3x+3$

$-2x = 4$

$\boxed{x = -2}$

c) $\left(\frac{1}{4}\right)2^{3x-1} = 4^{x-2}$

$2^{-2}2^{3x-1} = (2^2)^{x-2}$

$2^{3x-3} = 2^{2x-4}$

$3x-3 = 2x-4$

$\boxed{x = -1}$

2. Move the terms around to isolate the exponential on one side of the equation. Rewrite as an equivalent log equation, or take the log of both sides of the equation.

General

Ex 2: Solve for t.

a) $3^{2t} = 84$

$\Leftrightarrow \log_3 84 = 2t$

$\boxed{t = \frac{1}{2}\log_3 84}$

OR

$3^{2t} = 84$

$\ln 3^{2t} = \ln 84$

$2t \ln 3 = \ln 84$

$t = \frac{\ln 84}{2\ln 3} = \boxed{\frac{1}{2}\left[\frac{\ln 84}{\ln 3}\right]}$

b) $2(3^{2t-1}) - 5 = 11$

$2(3^{2t-1}) = 16$

$3^{2t-1} = 8$

$\Leftrightarrow \log_3 8 = 2t-1$

$2t = 1 + \log_3 8$

$\boxed{t = \frac{1}{2}(1 + \log_3 8)}$

c) $5e^{t-1} = 9$

$e^{t-1} = \frac{9}{5}$

$\Leftrightarrow \ln\left(\frac{9}{5}\right) = t-1$

$\boxed{t = 1 + \ln\left(\frac{9}{5}\right)}$

3. If there is an exponential term on both sides of the equation, you need to take the log of both sides.

Ex 3: Solve for t. $5^{1-t} = 12^t$

$\ln 5^{1-t} = \ln 12^t$

$(1-t)\ln 5 = t \ln 12$

$\ln 5 - (\ln 5)t = (\ln 12)t$

$\ln 5 = (\ln 5 + \ln 12)t$

$\ln 5 = (\ln 60)t$

$\boxed{\frac{\ln 5}{\ln 60} = t}$

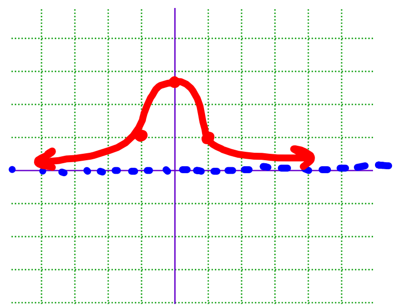
WARNING:

$\frac{\ln 5}{\ln 60} \neq \ln\left(\frac{5}{60}\right)$

Graphing an Exponential Function

Ex 4: Sketch this function by following these steps. $f(x) = e^{(1-x^2)}$

a) Determine the domain.



$x \in \mathbb{R}$

(or $(-\infty, \infty)$)

b) Find the *x*- and *y*- intercepts.

y-int: $y = e^{1-0} = e$

$\boxed{(0, e)}$

x-int: $0 \neq e^{1-x^2}$

$\boxed{\text{none}}$

c) Sketch any horizontal or vertical asymptotes

no VA (because no domain restriction)

HA: what happens to y-values when x is huge? $\boxed{y=0}$

d) Identify a few other points.

$y = e^{1-x^2}$

x	y
± 1	1
± 2	e^{-3}

NOTE: this is an even function because $f(x) = f(-x)$.

Application of Exponential Functions

Ex 5: A certain bacteria exhibits a growth according to this equation,

$P = 2000e^{2.5t}$ where P is the number present after t hours and the initial number is 2000.

a) How long does it take the population to double? $t = ?$ when $P = 4000$

$4000 = 2000e^{2.5t}$

$2 = e^{2.5t}$

$\ln 2 = 2.5t$

$\rightarrow t = \frac{\ln 2}{2.5} \simeq 0.27726$

$\Rightarrow$ after 0.27726 of an hour, bacteria has doubled.

b) When will the population reach 10,000?

$t = ?$ when $P = 10000$

$10000 = 2000e^{2.5t}$

$5 = e^{2.5t}$

$\ln 5 = 2.5t$

$t = \frac{\ln 5}{2.5} \simeq 0.6438$

$\Rightarrow$ after 0.6438 of an hour, population of bacteria has grown to be 10,000