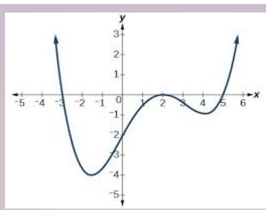
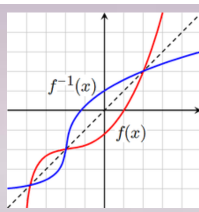
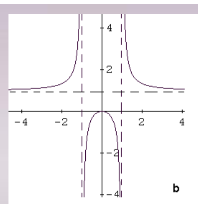
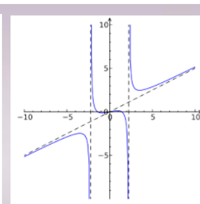
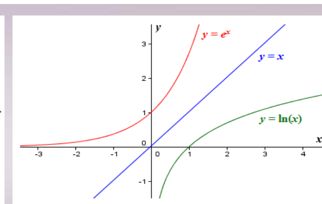










Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

17 Properties of Logarithms

Learning Objectives

- Use the definition of common and natural logarithms in solving equations and simplifying expressions.
- Use the change of base property to evaluate logarithms.
- Solve exponential equations using logarithmic properties.
- Combine and/or expand logarithmic expressions.
- Solve basic logarithmic equations using properties of logarithms and exponentials.

Common and Natural Logarithms

Base 10 is commonly used in logarithms. Thus, when no base is indicated, it is assumed to be base 10.

$\boxed{\log x = \log_{10} x}$

Ex 1: Evaluate these.

a) $\log 1{,}000{,}000$

$= \log_{10} 10^6 = 6$

b) $\log (10^{-3})$

$= -3$

c) $\log 0.01$

$= \log 10^{-2}$

$= -2$

d) $\log$ (a trillion)

$= \log (10^{12})$

$= 12$

Another base is the irrational number, *e,* called the natural base. This is written using $\ln x$.

$\boxed{\log_e x = \ln x}$

Ex 2: Evaluate these.

a) $\ln e$

$= \log_e (e^1)$

$= 1$

b) $\ln e^{-3}$

$= -3$

c) $\ln e^8$

$= 8$

d) $\ln\left(\frac{1}{e^5}\right)$

$= \ln (e^{-5})$

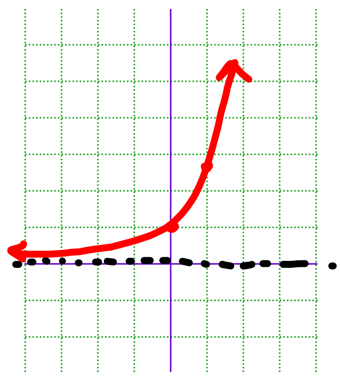
$= -5$

Natural Exponential Base

e ≈ 2.718281828459....

Ex 4: Sketch a graph of $y = e^x$.

(because $e>1$, it's exp. growth)



x	y
0	1
1	e
-100	really small

HA: $y=0$

The exponential base is used in financial and scientific calculations which we will explore in a later chapter.

Logarithm Properties

Let b be a positive number, not equal to 1, and let x be a positive number.

$\log_b 1 = 0$ $\Leftrightarrow$ $b^0 = 1$

$\log_b b = 1$ $\Leftrightarrow$ $b^1 = b$

$\log_b b^x = x$ $\Leftrightarrow$ $b^x = b^x$

If $\log_b x = \log_b y$, then $x = y$

(because log is one-to-one)

Ex 5: Evaluate these.

a) $\ln 1 = 0$

b) $\log 100$ $= \log 10^2 = 2$

c) $\ln e^{\pi} = \pi$

d) $\log(10^{0.2})$ $= 0.2$

Ex 6: Determine the value of x for each of these.

a) $\log x = \log(y + 5)$

$x = y + 5$

b) $\ln x = \ln(\pi + 1)$

$x = \pi + 1$

Properties of Logarithms

Change of Base Property

Let a and b be positive numbers, not equal to 1, and let x be a positive number.

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Ex 7: True or false? $\log_2 3 = \frac{\log 3}{\log 2}$

$b=2,\ a=10,\ x=3$

aside

$\log_2 3 = \frac{\ln 3}{\ln 2}$

True

Ex 8: Use your calculator to give an approximate value for these.

a) $\log_2 5$ $= \frac{\log 5}{\log 2} \approx 2.3219$

b) $\log 50$ ≈ 1.69897

c) $\ln 8$ ≈ 2.0794

d) $\log_6 0.0002$ $= \frac{\ln 0.0002}{\ln 6} \approx -4.7535$

Inverse Properties

Let b be a positive number, not equal to 1.

$b^{\log_b x} = x$, for any positive number x

$\log_b b^x = x$, for any real number x

logs & exponentials w/ same base undo each other.

Ex 9: Use the inverse properties to simplify.

a) $\ln e - 2$
$= (\ln e) - 2$
$= 1 - 2 = -1$

b) $\log_5 1 = 0$
$(\log_5 5^0 = 0)$

c) $6^{\log_6 20}$
$= 20$

d) $\log_3 3^{10}$
$= 10$

Exponent, Sum and Difference Properties of Logarithms

Let b be a positive number, not equal to 1, and let u and v be positive numbers.

① $\log_b(uv) = \log_b u + \log_b v$

② $\log_b\left(\frac{u}{v}\right) = \log_b u - \log_b v$

③ $\log_b x^m = m\log_b x$

WARNING:

$\log_b(u+v) \neq \log_b u + \log_b v$

Ex 10: Use these properties to expand these expressions.

a) $\log\sqrt{x^2(x+2)}$

$= \log\left(x^2(x+2)\right)^{1/2} \overset{③}{=} \frac{1}{2}\log\left(x^2(x+2)\right)$

$\overset{①}{=} \frac{1}{2}\left[\log x^2 + \log(x+2)\right] \overset{③}{=} \boxed{\frac{1}{2}\left[2\log x + \log(x+2)\right]}$

b) $\ln\left(\frac{x^2-1}{x^3}\right), \quad x > 1$

$\overset{②}{=} \ln(x^2-1) - \ln(x^3)$

$\overset{③}{=} \boxed{\ln(x^2-1) - 3\ln x}$

Ex 11: Use these properties to contract these expressions into a single term.

a) $3\log x + 4\log y - 5\log z$

$\overset{③}{=} \log x^3 + \log y^4 - \log z^5$

$\overset{①}{=} \log(x^3y^4) - \log z^5$

$\overset{②}{=} \boxed{\log\left(\frac{x^3y^4}{z^5}\right)}$

b) $\frac{1}{2}[\ln(x+1) + 2\ln(x-1)] - 6\ln x$

$\overset{③}{=} \frac{1}{2}\left[\ln(x+1) + \ln(x-1)^2\right] - \ln x^6$

$\overset{①}{=} \frac{1}{2}\ln\left[(x+1)(x-1)^2\right] - \ln x^6$

$\overset{③}{=} \ln\left[(x+1)(x-1)^2\right]^{1/2} - \ln x^6$

$\overset{②}{=} \ln\left[\frac{\left((x+1)(x-1)^2\right)^{1/2}}{x^6}\right]$

$= \boxed{\ln\left[\frac{\sqrt{(x+1)(x-1)^2}}{x^6}\right]}$