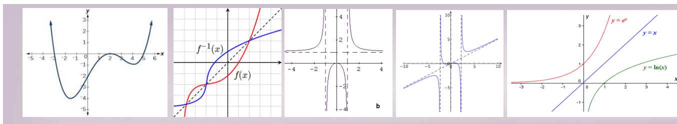


Math 1050 ~ College Algebra

16 Introduction to Exponentials and Logarithms

$$-3x+4y=5$$
$$2x-y=-10$$
$$\begin{bmatrix}-3 & 4\\ 2 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ -10\end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1-z^{n+1}}{1-z}$$

Learning Objectives

- Evaluate exponential expressions and functions.
- Graph basic exponential functions, including transformations.
- Use the one-to-one property to solve common-base exponential equations.
- Evaluate logarithmic expressions and functions.
- Solve logarithmic equations by conversion to exponential form.
- Graph basic logarithmic functions, including transformations.

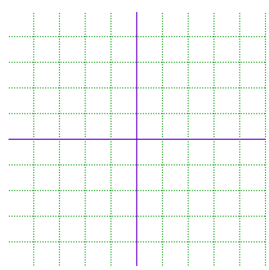
Definition of an Exponential Function

An **exponential function** is one in which the variable is in the exponent.

$f(x) = b^x$, where $b > 0$, $b \neq 1$, $x \in \mathcal{R}$

Ex 1: Fill out the table and plot the graph of $y = 2^x$.

x	$f(x)$	$(x, f(x))$
-3		
-1		
0		
1		
2		



Notice these things about the graph above.

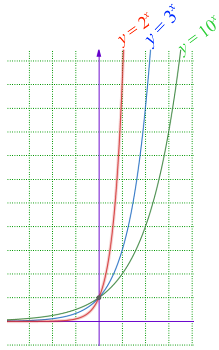
Domain Range y-intercept

Horizontal asymptote

Exponential growth

Horizontal line test

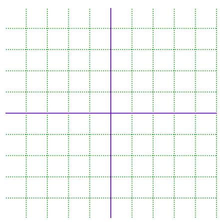
As the base, b changes note how little else does.



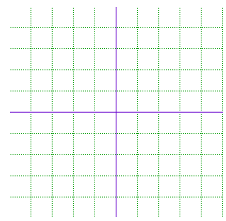
We can use transformations learned previously to graph variations.

Ex 2: Use transformations to sketch these functions.

a) $f(x) = 3^{(x-1)} + 2$



b) $g(x) = 2^{(3-x)} - 1$



<u>Definition of a Logarithm</u>

For $y > 0$ and b a positive constant other than 1, $\log_b y$ is called a <u>logarithm</u> in base b of y, and is the power of b that gives y.

$y = \log_b x \Leftrightarrow x = b^y$

Ex 3: Find the exact value for each of these.

a) $\log_2 16$ b) $\log_{10} 100000$ c) $\log_5 \frac{1}{125}$ d) $\log_8 4$

Ex 4: Convert from logarithmic form to exponential form or visa versa.

a) $9^{3/2} = 27$ b) $\log_8 \sqrt{8} = \frac{1}{2}$ c) $\log_{32} 4 = \frac{2}{5}$ d) $10^{-3} = 0.001$

To solve a logarithmic equation, it is convenient to turn it into an exponential equation.

Ex 5: Solve each equation.

a) $\log_2(x-1) = 5$

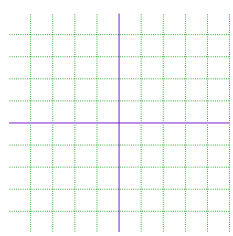
b) $\log_{10}(3z) = 2$

Definition of a Logarithmic Function

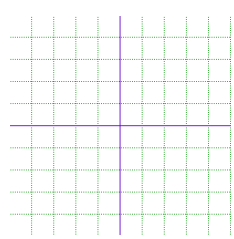
$f(x) = \log_b x$ is a logarithmic function with $x > 0$, $b > 0$ and $b \neq 1$.

Ex 6: Fill in the table and sketch a graph of $f(x) = \log_2 x$

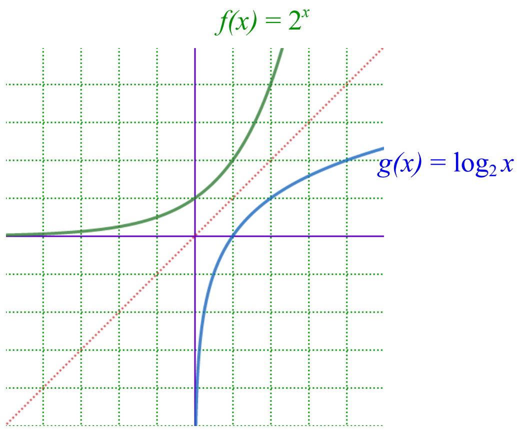
x	$f(x)$	$(x, f(x))$
1/4		
1/2		
1		
2		
4		



Ex 7: Use transformations to sketch $f(x) = -\log_2(x) + 1$



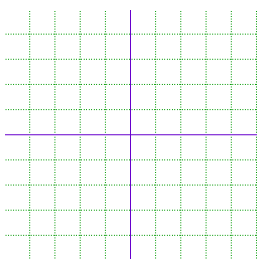
Relationship of Exponential and Logarithmic Functions



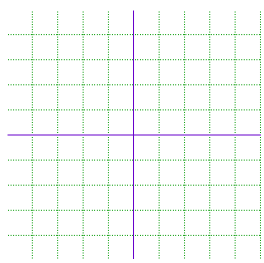
Ex 7: Note the symmetry in the two functions. Compute this.

$(g \circ f)(x) =$

Ex 8: All of the previous graphs given in this lesson have the characteristic that $b > 1$. Examine what happens when $0 < b < 1$. Sketch below for $b = ½$.



$f(x) = b^x,\ b = ½$



$f(x) = \log_b x,\ b = ½$

Properties of Graphs of Logarithmic And Exponential Functions