Math 1050 ~ College Algebra

16 Introduction to Exponential and Logarithms

Learning Objectives

- Evaluate exponential expressions and functions.
- Graph basic exponential functions, including transformations.
- Use the one-to-one property to solve common-base exponential equations.
- Evaluate logarithmic expressions and functions.
- Solve logarithmic equations by conversion to exponential form.
- Graph basic logarithmic functions, including transformations.

Definition of an Exponential Function

An exponential function is one in which the variable is in the exponent.

\[ f(x) = b^x \], where \( b > 0, b \neq 1, x \in \mathbb{R} \]

Ex 1: Fill out the table and plot the graph of \( y = 2^x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( (x, f(x)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>( 2^{-3} )</td>
<td>( (-3, \frac{1}{8}) )</td>
</tr>
<tr>
<td>-1</td>
<td>( 2^{-1} )</td>
<td>( (-1, \frac{1}{2}) )</td>
</tr>
<tr>
<td>0</td>
<td>( 2^0 )</td>
<td>( (0, 1) )</td>
</tr>
<tr>
<td>1</td>
<td>( 2^1 )</td>
<td>( (1, 2) )</td>
</tr>
<tr>
<td>2</td>
<td>( 2^2 )</td>
<td>( (2, 4) )</td>
</tr>
</tbody>
</table>

Notice these things about the graph above.

- Domain: \( \mathbb{R} \)
- Range: \( (0, \infty) \)
- \( y \)-intercept: \( (0, 1) \)
- Horizontal asymptote
- Exponential growth

Horizontal line test
As the base, \( b \) changes note how little else does.

We can use transformations learned previously to graph variations.

Ex 2: Use transformations to sketch these functions.

a) \( f(x) = 3^{(x-1)} + 2 \)

b) \( g(x) = 2^{(3-x)} - 1 \)

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**Definition of a Logarithm**

For \( y > 0 \) and \( b \) a positive constant other than 1, \( \log_b y \) is called a *logarithm* in base \( b \) of \( y \), and is the power of \( b \) that gives \( y \).

\[
y = \log_b x \iff x = b^y
\]

Ex 3: Find the exact value for each of these.

a) \( \log_2 16 \)  
   b) \( \log_{10} 100000 \)  
   c) \( \log_5 \frac{1}{125} \)  
   d) \( \log_8 4 \)

Ex 4: Convert from logarithmic form to exponential form or visa versa.

a) \( 9^{\frac{1}{2}} = 27 \)  
   b) \( \log_8 \sqrt{8} = \frac{1}{2} \)  
   c) \( \log_5 4 = \frac{2}{5} \)  
   d) \( 10^{-3} = 0.001 \)
To solve a logarithmic equation, it is convenient to turn it into an exponential equation.

Ex 5: Solve each equation.

a) \( \log_2(x-1) = 5 \)  

b) \( \log_{10}(3z) = 2 \)

**Definition of a Logarithmic Function**

\( f(x) = \log_b x \) is a logarithmic function with \( x > 0, b > 0 \) and \( b \neq 1 \).

Ex 6: Fill in the table and sketch a graph of \( f(x) = \log_2 x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( (x, f(x)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ex 7: Use transformations to sketch \( f(x) = -\log_2(x) + 1 \)
**Relationship of Exponential and Logarithmic Functions**

![Graph of exponential and logarithmic functions]

Ex 7: Note the symmetry in the two functions. Compute this.

\[(g \circ f)(x) = \]

Ex 8: All of the previous graphs given in this lesson have the characteristic that \( b > 1 \). Examine what happens when \( 0 < b < 1 \). Sketch below for \( b = \frac{1}{2} \).

\[f(x) = b^x, \ b = \frac{1}{2}\]

\[f(x) = \log_b x, \ b = \frac{1}{2}\]

**Properties of Graphs of Logarithmic And Exponential Functions**