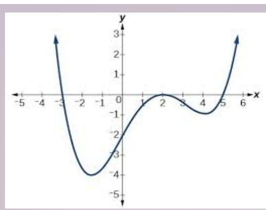
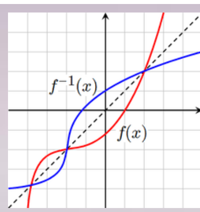
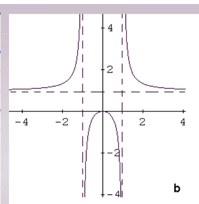
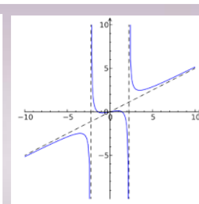
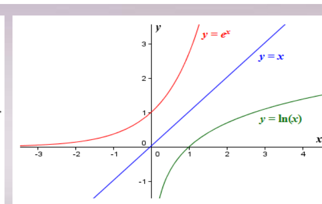










Math 1050 ~ College Algebra

16 Introduction to Exponentials and Logarithms

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

Learning Objectives

- Evaluate exponential expressions and functions.
- Graph basic exponential functions, including transformations.
- Use the one-to-one property to solve common-base exponential equations.
- Evaluate logarithmic expressions and functions.
- Solve logarithmic equations by conversion to exponential form.
- Graph basic logarithmic functions, including transformations.

Definition of an Exponential Function

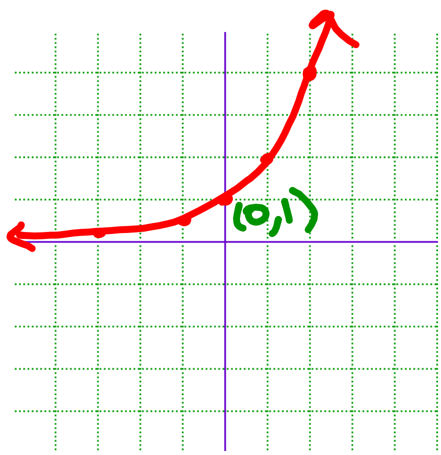
An **exponential function** is one in which the variable is in the exponent.

$\boxed{f(x) = b^x}$, where $b > 0$, $b \neq 1$, $x \in \mathcal{R}$ (b is called the base)

Ex 1: Fill out the table and plot the graph of $y = 2^x$.

x	$f(x)$	$(x, f(x))$
-3	$2^{-3} = 1/8$	$(-3, 1/8)$
-1	$2^{-1} = 1/2$	$(-1, 1/2)$
0	$2^0 = 1$	$(0, 1)$
1	$2^1 = 2$	$(1, 2)$
2	$2^2 = 4$	$(2, 4)$

$y = 2^x$



Notice these things about the graph above.

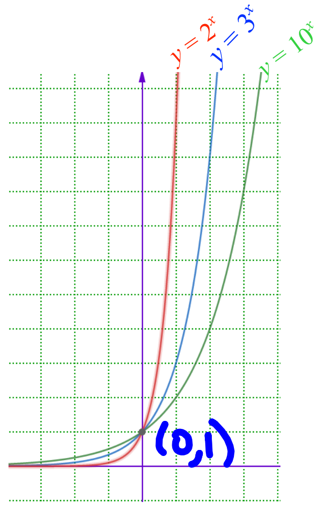
Domain $(-\infty, \infty)$ Range $(0, \infty)$ y-intercept $(0, 1)$

Horizontal asymptote $y = 0$ (note: this exponential fn has HA that only describes left behavior, not right-side behavior)

Exponential growth
- describes right-side behavior
- y-values are always increasing

Horizontal line test
this graph passes horiz. line test ⇒ it's one-to-one, (i.e. it has an inverse)

As the base, b changes note how little else does.



- shape of curves is about the same
- they all have y-intercept of (0,1)
- they all have HA $y=0$
- exponential growth

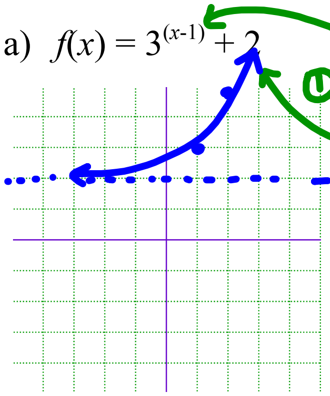
We can use transformations learned previously to graph variations.

Ex 2: Use transformations to sketch these functions.

a) $f(x) = 3^{(x-1)} + 2$

① shift R 1

② shift up 2



base graph $y=3^x$	$f(x)=y$
(0,1)	(1,3)
(1,3)	(2,5)

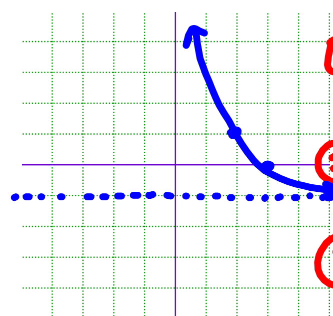
HA: $y=2$

b) $g(x) = 2^{(3-x)} - 1 = 2^{-(x-3)} - 1$

① horiz. reflectn

② shift R 3

③ shift down 1



$y=2^x$	$y=2^{-x}$	$y=2^{-(x-3)}-1$
(0,1)	(0,1)	(3,0)
(1,2)	(-1,2)	(2,1)

HA: $y=-1$

Definition of a Logarithm

For $y > 0$ and b a positive constant other than 1, $\log_b y$ is called a <u>logarithm</u> in base b of y, and is the power of b that gives y.

↳ read "log base b of y"

$\boxed{y = \log_b x \Leftrightarrow x = b^y}$

related ex: $15 \div 3 = 5 \Longleftrightarrow 15 = 3 \cdot 5$

Ex 3: Find the exact value for each of these.

a) $\log_2 16 = ?$

$2^? = 16$

4

b) $\log_{10} 100000$

$= \log_{10} 10^5$

5

c) $\log_5 \frac{1}{125}$

$= \log_5 (5^{-3})$

-3

d) $\log_8 4 = ?$

$\frac{2}{3}$

$8^? = 4$

$= 2^2 = (2^3)^{2/3}$

Ex 4: Convert from logarithmic form to exponential form or visa versa.

a) $9^{3/2} = 27$

$b = 9,\ y = \frac{3}{2},\ x = 27$

$\boxed{\frac{3}{2} = \log_9 27}$

b) $\log_8 \sqrt{8} = \frac{1}{2}$

$b = 8,\ x = \sqrt{8},\ y = \frac{1}{2}$

$\boxed{\sqrt{8} = 8^{1/2}}$

c) $\log_{32} 4 = \frac{2}{5}$

$\boxed{32^{2/5} = 4}$

d) $10^{-3} = 0.001$

$\boxed{\log_{10} 0.001 = -3}$

To solve a logarithmic equation, it is convenient to turn it into an exponential equation.

use defn of log:

$\log_b \heartsuit = \bigstar \Leftrightarrow b^{\bigstar} = \heartsuit$

Ex 5: Solve each equation.

a) $\log_2(x\text{-}1) = 5$

$2^5 = x-1$

$32+1 = x$

$\boxed{x=33}$

b) $\log_{10}(3z) = 2$

$10^2 = 3z$

$\boxed{\frac{100}{3} = z}$

Definition of a Logarithmic Function

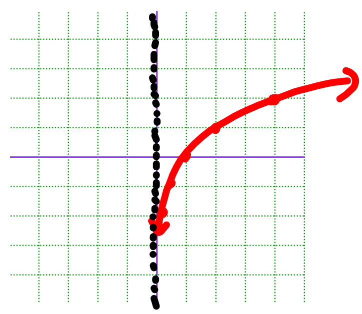
$\boxed{f(x) = \log_b x}$ is a logarithmic function with $x > 0$, $b > 0$ and $b \neq 1$.

domain

Ex 6: Fill in the table and sketch a graph of $f(x) = \log_2 x$

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

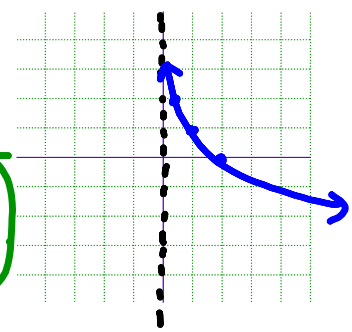
x	$f(x)$	$(x, f(x))$
1/4	$\log_2(1/4) = -2$	$(1/4, -2)$
1/2	$\log_2(1/2) = -1$	$(1/2, -1)$
1	$\log_2(1) = 0$	$(1, 0)$
2	$\log_2(2) = 1$	$(2, 1)$
4	$\log_2(4) = 2$	$(4, 2)$



VA: $x=0$

Ex 7: Use transformations to sketch $f(x) = -\log_2(x) + 1$

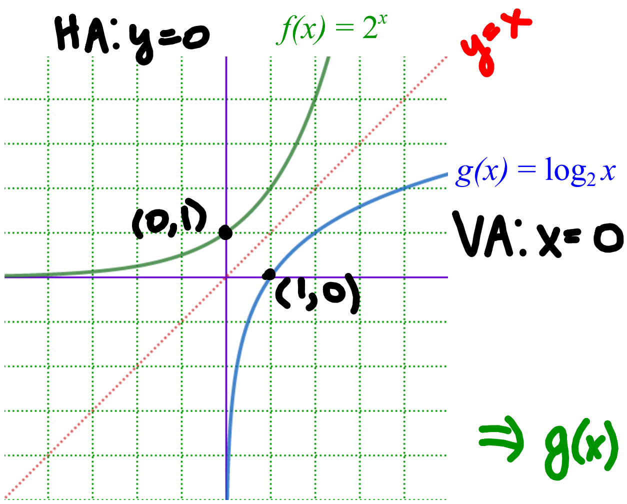
① vert. ref
② shift up 1



$y=\log_2 x$	$y=-\log_2(x)$	$y=f(x)$
$(\frac{1}{2}, -1)$	$(\frac{1}{2}, 1)$	$(\frac{1}{2}, 2)$
$(1, 0)$	$(1, 0)$	$(1, 1)$
$(2, 1)$	$(2, -1)$	$(2, 0)$

VA: $x=0$

Relationship of Exponential and Logarithmic Functions



Ex 7: Note the symmetry in the two functions. Compute this.

$$(g\circ f)(x) = g(f(x))$$
$$= g(2^x)$$
$$= \log_2(2^x)$$
$$= x$$

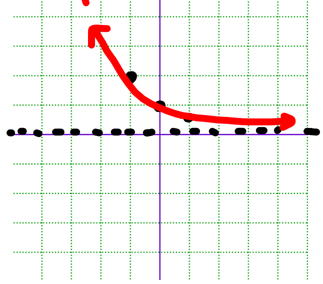
$\Rightarrow$ g(x) & f(x) are inverse fns!

Ex 8: All of the previous graphs given in this lesson have the characteristic that $b > 1$. Examine what happens when $0 < b < 1$. Sketch below for $b = ½$.

$y=\left(\frac{1}{2}\right)^x$

x	y
-1	2
0	1
1	½

exponential decay

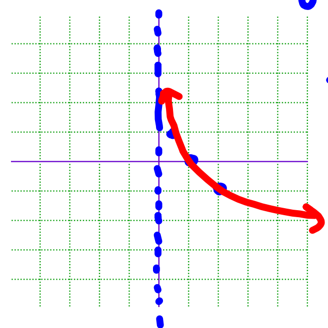


$f(x) = b^x,\ b = ½$

HA: $y=0$

$y=\log_{\frac{1}{2}} x$

x	y
$\frac{1}{2}$	1
1	0
2	-1



$f(x) = \log_b x,\ b = ½$

VA: $x=0$

Properties of Graphs of Logarithmic and Exponential Functions

	exp	log
pt :	(0,1)	(1,0)
asymptote:	HA $y=0$	VA $x=0$
$b>1$:	increasing	increasing
$0<b<1$:	decreasing	decreasing