Math 1050 ~ College Algebra

14 Graphs with Holes and Variations on Asymptotes

Learning Objectives

- Identify holes in the graph of a rational function.
- Graph rational functions without vertical asymptotes.
- Find slant (oblique) asymptotes.
- Graph rational functions having slant asymptotes.

\[
\begin{align*}
-3x + 4y &= 5 \\
2x - y &= -10 \\
\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 5 \\ -10 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\sum_{k=1}^{m} k &= \frac{m(m + 1)}{2} \\
\sum_{k=0}^{n} z^k &= \frac{1 - z^{n+1}}{1 - z}
\end{align*}
\]
Since there can be no points on the vertical asymptotes, what happens in an example like this?

**Note:** To graph a rational fn, add to your steps to (1) simplify completely and (2) find all holes (before finding VA, etc.)

Ex 1: Analyze and graph.

\[
H(x) = \frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)}
\]

\[
H(x) = \frac{1}{x+2}
\]

We lost info that \(x \neq 2\)

Hole: \((2, \frac{1}{4})\) \(y = \frac{1}{2+2}\)

VA: \(x = -2\)
Graphing Rational Functions with No Vertical Asymptotes

Ex 2: Analyze and graph $H(x) = \frac{2x + 3}{x^3 + 2}$

- **VA:** none
- **HA:** $y = 0$
- **No holes**
- **x-int:** $D = 2x + 3$  
  $x = -\frac{3}{2}$  
  $(-\frac{3}{2}, 0)$
- **y-int:** $y = \frac{0 + 3}{0 + 2} = \frac{3}{2}$  
  $(0, \frac{3}{2})$

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\[ -\frac{3}{2} \rightarrow \text{sign line} \rightarrow + \]
Identifying Slant (Oblique) Asymptotes

Strategy to find SA:
1. look for HA first. If there is NO HA, then
2. do long division (i.e. divide denominator of rational fn into its numerator)
3. y = result of division WITHOUT remainder is SA.

NOTE: we only look for SA when degree of numerator > degree of denominator.

Ex 3: Analyze and graph.

\[ H(x) = \frac{x^2 - x + 6}{x + 3} \]

VA: \( x = -3 \)

HA: none

need to look for SA.

\[
\begin{array}{c|c}
  \text{long division} & \text{numerator does not factor} \\
  \hline
  x - 4 & D = x^2 - x + 6 \\
  x + 3 & \text{N.S.} \\
  \hline
  -4x + 6 & x - 4 \\
  -(-4x - 12) & \\
  \hline
  & \frac{18}{x + 3}
\end{array}
\]

\( \Rightarrow \) SA: \( y = x - 4 \)
Ex 4: Analyze and graph.

\[ f(x) = \frac{x^3 - 1}{x - 1} \]

\[ f(x) = (x-1)(x^2 + x + 1) \]  

\[ f(x) = x^2 + x + 1 \]

hole: \((1, 3)\)

\[ y = 1^2 + 1 + 1 = 3 \]

no VA

no HA

vertex:

\[ x = \frac{-1}{2(1)} = -\frac{1}{2} \]

\[ (-\frac{1}{2}, \frac{3}{4}) \]

\[ (\frac{3}{4})^2 + \frac{1}{2} + 1 = \frac{9}{16} + \frac{1}{2} \]

\[ = \frac{3}{4} \]