Math 1050 ~ College Algebra

13 Graphing Rational Functions

Learning Objectives

- Graph irreducible rational functions with denominators of degree greater than one and numerators having the same or a lesser degree.

\[
\begin{align*}
-3x + 4y &= 5 \\
2x - y &= -10 \\
\begin{bmatrix}
-3 & 4 \\
2 & -1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
&= 
\begin{bmatrix}
5 \\
-10
\end{bmatrix}
\end{align*}
\]

\[
\sum_{k=1}^{m} k = \frac{m(m + 1)}{2}
\]

\[
\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}
\]
In our toolkit of functions, we have two rational functions.

Ex 1: Sketch these using transformations of the toolkit function.

a) \( g(x) = \frac{4}{x+1} - 2 \)

b) \( f(x) = \frac{1}{(x-2)^2} + 3 \)

Not all rational functions can be put in this form. It is helpful to follow the steps in the previous lesson to get a graph of a rational function.

1. Determine the domain and plot vertical asymptotes.
2. Find and plot the x- and y- intercepts.
3. Determine and plot the end-behavior asymptotes.
4. Use a sign-line and the value of other points to complete the graph.
Ex 2: For each of these, determine the x and y-intercepts, vertical and horizontal asymptotes and sketch a graph.

a) \( f(x) = \frac{3}{1-x} \)
   - **VA:** \( x = 1 \)
   - **HA:** \( y = 0 \)
   - x-int: \( 0 = \frac{3}{1-x} \) \( \Rightarrow \) \( x = \frac{3}{1} = 3 \)
   - y-int: \( x = 0 \) \( \Rightarrow \) \( y = \frac{3}{1} = 3 \)

b) \( g(x) = \frac{3-x}{x^2+4} \)
   - **VA:** None
   - **HA:** \( y = 0 \)
   - x-int: \( 0 = \frac{3-x}{x^2+4} \)
     - \( x = 3 \)
   - y-int: \( x = 0 \) \( \Rightarrow \) \( y = \frac{3}{4} \)

h) \( h(x) = \frac{2x^2 - 5x - 3}{x^2 + x - 2} \)
   - **VA:** \( x = -2, x = 1 \)
   - **HA:** None
   - x-int: \( 2x^2 - 5x - 3 = 0 \)
     - \( x = -\frac{1}{2}, 3 \)
   - y-int: \( x = 0 \) \( \Rightarrow \) \( y = \frac{2}{1} = 2 \)
Ex 3: Analyze and graph.

\[ f(x) = \frac{(x-4)(x-2)^2}{(x+3)^2(x-1)} \]

**VA:** \( x = 1, x = 3 \)

**HA:** end behavior \( y = 1 \)

is like \( \frac{x(x^2)}{x^3} = \frac{x}{x^3} = 1 \)

x-int: \( 0 = (x-4)(x-2)^2 \)

\( x = 4, 2 \)

\( (4,0), (2,0) \)

y-int: \( y = \frac{(0-4)(0-2)^2}{3^2(-1)} = -\frac{16}{-9} = \frac{16}{9} \)

\( (0, \frac{16}{9}) \)

\((3,?)\)