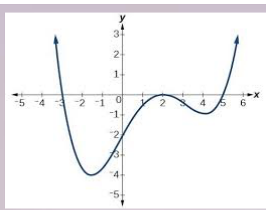
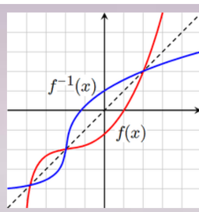
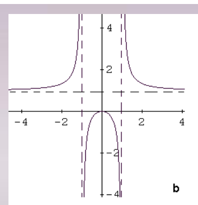
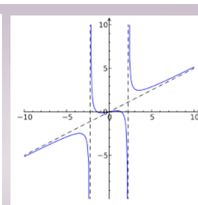
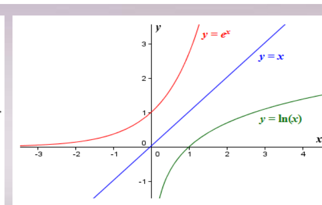



Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

12 Introduction to Rational Functions

Learning Objectives

- **Identify a rational function.**
- **Determine the domain of a rational function.**
- **Find the *x*- and *y*-intercepts for a rational function.**
- **Identify vertical and horizontal asymptotes.**
- **Graph irreducible rational functions with constant or first degree numerators and denominators of degree one.**

A <u>rational function</u> is a ratio of two polynomial functions.

$f(x)=\frac{N(x)}{D(x)}$ where N(x) and D(x) are polynomials.

N(x) = numerator polynomial

D(x) = denominator polynomial

Note: all polynomials are a subset of rational fns.

Ex 1: Determine which of these functions are rational functions.

a) $f(x)=\frac{x^2+1}{x+4}$

both N(x) & D(x) are polynomials

Yes

b) $f(x)=\frac{3x+2}{\sqrt{x}-3}$

D(x) is NOT a polynomial

No

c) $f(x)=\frac{x^2-2x-3}{\pi}$

N(x) is 2nd deg polynomial

D(x) is a 0 deg polynomial

Yes

d) $f(x)=\frac{x^{2.5}+5}{x^2-25}$

N(x) is NOT a polynomial

No

<u>Vertical Asymptotes of Simplified Rational Functions</u>

- determined by finding disallowed denominator values
- line $\boxed{x=a}$ where D(a) = 0
- graph will <u>never</u> cross or touch

(i.e. x-values that make D(x) = 0)

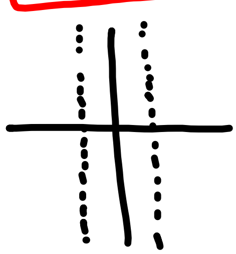
Ex 2: Find the domain and the vertical asymptotes for these functions.

a) $f(x)=\frac{2x^2}{x^2-1}$

$f(x)=\frac{2x^2}{(x-1)(x+1)}$

$x\neq 1,-1$ domain

VA: $\boxed{x=1 \text{ and } x=-1}$



b) $f(x)=\frac{3x}{x^2+1}$

domain: $x\in\mathbb{R}$ (or $(-\infty,\infty)$)

there are no VA.

c) $f(x)=\frac{x+4}{4x-2x^2}=\frac{x+4}{2x(2-x)}$

domain: $x\neq 0,2$

$(-\infty,0)\cup(0,2)\cup(2,\infty)$

VA: $\boxed{x=0,\ x=2}$

Horizontal Asymptotes

(describe "end" behavior of graph, i.e. far left and/or right behavior)

- end behavior of the graph
- line $\boxed{y = b}$ where $\lim_{x \to \pm\infty} f(x) = b$ y-value of f(x) gets super close to b as x gets huge (+ or −)
- graph may cross it
- depends on degree of N(x) and D(x)
  - degree (N(x)) < degree (D(x)), $\boxed{y = 0 \text{ HA}}$
  - degree (N(x)) = degree (D(x)), y = ratio of the leading coefficients. HA

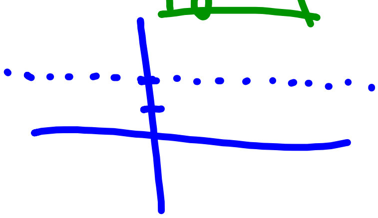
Ex 3: Find the horizontal asymptotes of these functions.

a) $f(x) = \dfrac{2x^2}{x^2 - 1}$

as x gets really huge, what matters most is

$$\frac{2x^2}{x^2} = 2$$

$\Rightarrow$ HA: $\boxed{y=2}$



b) $f(x) = \dfrac{3x}{x^2 + 1}$

as x gets really huge, this behaves similar to $\dfrac{3x}{x^2} = \dfrac{3}{x}$

(note: thinking is that $\dfrac{3}{\text{super huge}}$ = super small)

$\Rightarrow$ HA: $\boxed{y=0}$

c) $f(x) = \dfrac{x+4}{4x - 2x^2}$

HA: $\boxed{y=0}$

as x gets super huge, f(x) will eventually behave like $\dfrac{x}{-2x^2} = \dfrac{1}{-2x}$

$\to 0$

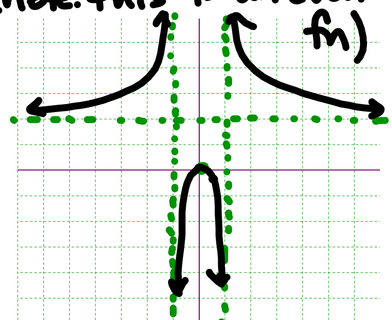
Ex 4: For each of these functions, determine the x and y-intercepts, vertical and horizontal asymptotes and draw a quick sketch.

a) $f(x)=\dfrac{2x^2}{x^2-1}$

VA: $x=1$, $x=-1$

HA: $y=2$

(note: this is an even fn)



y-int: $f(0)=\dfrac{0}{0-1}=0$

$\boxed{(0,0)}$

x-int: $\dfrac{2x^2}{x^2-1}=0$

$2x^2=0$

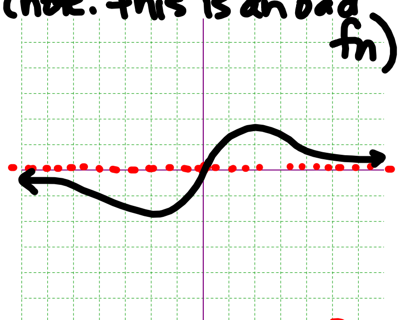
$x=0$

b) $f(x)=\dfrac{3x}{x^2+1}$

VA: none

HA: $y=0$

(note: this is an odd fn)



y-int: $f(0)=\dfrac{0}{0+1}=0$

$\boxed{(0,0)}$

x-int: $\dfrac{3x}{x^2+1}=0$

$3x=0$

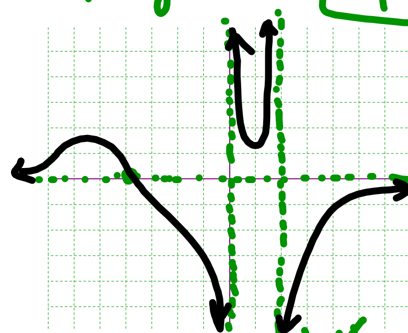
$x=0$

c) $f(x)=\dfrac{x+4}{4x-2x^2}$

x	y
-1	−
1	+
3	−

VA: $x=0$, $x=2$

HA: $y=0$



y-int: $f(0)=\cancel{\dfrac{4}{0}}$

$\boxed{\text{no y-int.}}$

x-int:

$\dfrac{x+4}{4x-2x^2}=0$

$x+4=0$

$x=-4$

$\boxed{(-4,0)}$