9.5 The Binomial Theorem

* Use the Binomial Theorem to calculate binomial coefficients.
* Use Pascal's Triangle to calculate binomial coefficients.
* Find the $n$th term in a binomial expansion.

What does the word *binomial* mean?

$$(a+b)^0$$
$$(a+b)^1$$
$$(a+b)^2$$
$$(a+b)^3$$
What does 7! mean?

Example 1: Determine the value of each of these.

a) 4!
b) 10!
c) 12!/10!
d) n!
e) (n+2)!
f) 0!

Example 2: A pizza shop offers 4 different toppings, Onions, Mushrooms, Pepperoni and Ham. How many 'different' pizzas can you order having none, one, two, three or all four toppings?

What does \( \binom{n}{r} \) mean?

\[ \binom{n}{r} = \frac{n!}{(n-r)!r!} = \binom{n}{r} \]

Determine the value of each of these.

\( \binom{4}{0} \) \( \binom{4}{1} \) \( \binom{4}{2} \) \( \binom{4}{3} \) \( \binom{4}{4} \)
Example 3: Determine the value of each of these and make up a question it might answer.

\[ \binom{3}{2} \]

\[ \binom{10}{8} \]

\[ \binom{10}{4} \]

\[ \binom{6}{0} \]

**Binomial Theorem** and Pascal's Triangle

\[ (x+y)^n = \]

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
1 & 5 & 10 & 10 & 5 & 1
\end{array}
\]

So, \((a+b)^5 = \)
Example 4: Expand this binomial. \((2x - y)^4 =

\[
\begin{array}{cccccc}
1 & \binom{4}{0} & \binom{4}{1} & \binom{4}{2} & \binom{4}{3} & \binom{4}{4} \\
1 & 4 & 6 & 4 & 1 \\
1 & 5 & 10 & 10 & 5 & 1 \\
1 & 6 & 15 & 20 & 15 & 6 & 1
\end{array}
\]

Example 5: How do we find the \(x^n\) term in the expansion of \((2x-y)^{10}\) without writing the entire expansion?
Example 6: An interesting application of Pascal's Triangle is in probability. In a family of six children, what is the probability that two are boys and the rest are girls?