9.2 Arithmetic Sequences and Series

In section 9.2 you will learn to:

- Recognize, write and find the nth terms of arithmetic sequences.
- Find the nth partial sums of arithmetic sequences.
- Use arithmetic sequences to model and solve real-life problems.

$1, 4, 7, 10, \ldots, 3n-2, \ldots$

$\sum_{i=1}^{3} 2i = 12$

applications

A sequence $a_1, a_2, a_3, \ldots, a_n$ is said to be *arithmetic* is the difference d between consecutive terms remains constant.

$a_k - a_{k-1} = d$

Which of these are arithmetic sequences?

1, 3, 5, 7, ... arithmetic

✓ 2, 4, 8, 16, ... not
(2, 4, 8)

1, 4, 9, 16, ... not
(3, 5, 7)

✓ 19, 11, 3, -5, -13, ... arith.
(-8, -8, -8)

Example 1:

Find the next three terms of the arithmetic sequence 1, 5, 9, 13, ... 17, 21, 25, ...

Then, find a formula for the n^{th} term and use that to calculate a_{100}.

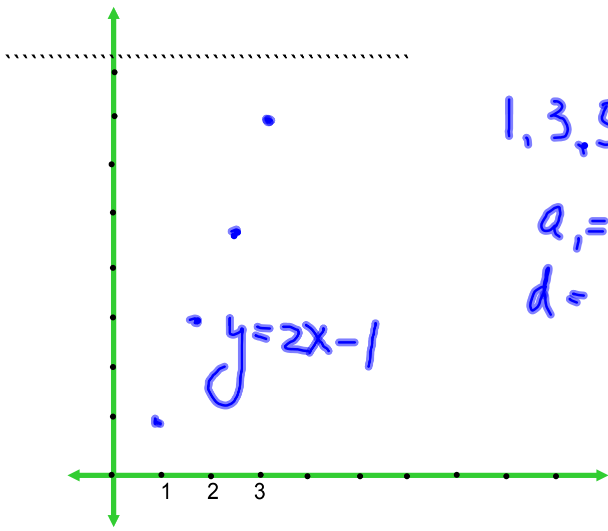
$a_1 = a = 1$

$d = 4$

$a_n = a_1 + (n-1)d$

$a_{100} = 1 + 99(4) = 1 + 396 = 397$

How would you describe the graph of an arithmetic sequence?



$1, 3, 5, 7, \ldots$

$\left.\begin{array}{l} a_1 = 1 \\ d = 2 \end{array}\right\} \Rightarrow a_n = 1 + (n-1)(2) =$

$= 2n - 1$

Example 2:

Suppose the 4th term of an arithmetic sequence is 20 and the 13th term is 65. What are the first six terms of the sequence?

$a_{13} - a_4 = 9d$

$65 - 20 = 9d$

$45 = 9d$

$5 = d$

$a_4 = a_1 + 3d$

$20 = a_1 + 3(5)$

$5 = a_1$

$\underline{5}, \underline{10}, \underline{15}, \underline{20}, \underline{25}, \underline{30}$

Example 3:

A local theatre has a large auditorium with 22 rows of seats. There are 18 seats on Row 1 and each row after Row 1 has two more seats than the previous row. How many seats are in Row 22?

$18, 20, 22, \ldots$

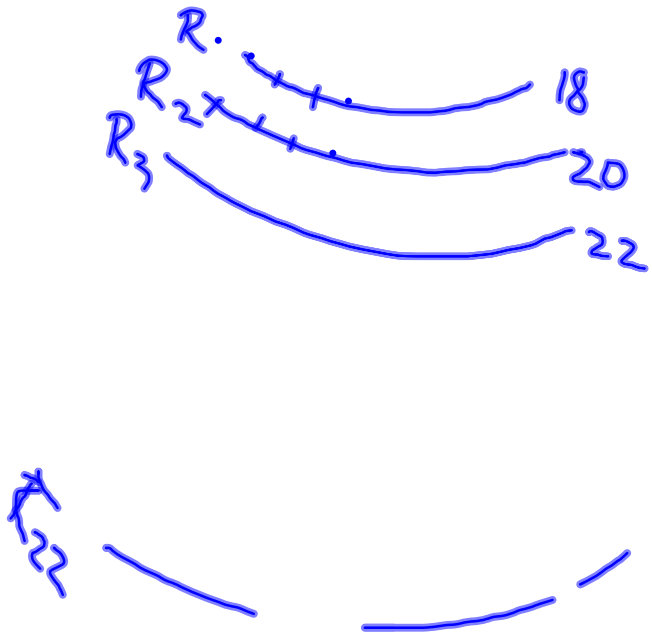
$a_1 = 18$

$d = 2$

$a_{22} = a_1 + (22-1)(2)$

$= 18 + 21(2)$

$R_{22} = 18 + 42 = 60$ seats



A *finite arithmetic series* is the sum S_n of the first n terms of a finite arithmetic sequence.

$$Sn = a_1 + (a_1+d) + (a_1 + 2d) + (a_1 + 3d) + ... + (a_1 + (n-1)d)$$

S_n can be found by computing $S_n = \frac{n}{2}(a_1 + a_n)$

$$S_n = a_1 + (a_1+d) + (a_1+2d) + \cdots + (a_1+(n-1)d)$$

$$S_n = (a_1+(n-1)d) + (a_1+(n-2)d) + \cdots + a_2 + a_1$$

$$2S_n = 2a_1+(n-1)d + (2a_1+(n-1)d) + (2a_1+(n-1)d) + \cdots + (2a_1+(n-1)d)$$

$$2S_n = n(2a+(n-1)d)$$

$$S_n = \frac{n}{2}(a_1 + a_1+(n-1)d) = \frac{n}{2}(a_1+a_n)$$

An alternate formula for S_n is $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

Example 4:

Use the summation notation to write these series and find the sums using either of the formulas:

$$S_n = \frac{n}{2}(a_1 + a_n)$$ ✓

or

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$ ✓✓

a) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17:

$a_1 \ldots a_9 = ?$

$$\sum_{i=1}^{9}(1+(i-1)(2)) = \frac{9}{2}(1+17) = \frac{9}{2}(18) = 81$$

b) $\sum_{j=1}^{30}(9+3j)$

$$= \frac{n}{2}(2a_1 + (n-1)d) =$$

$a_1 = 12$

$d = 3$

$n = 30$

$$= \frac{30}{2}(2(12)+(29)(3)) =$$

$$= 1665$$

Example 5:

In the theatre we described previously, there were 22 rows of seating. There were 18 seats on Row 1 and each subsequent row had two more seats than the previous row.

What is the seating capacity of the auditorium?

$R_1 = 18$

$R_{22} = 60$

$n = 22$

$S_{22} = \frac{22}{2}(18 + 60) =$

$= 11 \cdot 78 = 858$ seats

Example 6:

In the last lesson, you decided to save for your trip to Europe. You opened a savings account with $1.00 and on each subsequent day, you deposited a dollar more than on the previous day.

How much have you contributed by the end of one year?

$$1 + 2 + 3 + \cdot \cdot \cdot \cdot + 365 =$$

$$= \frac{365}{2}(1 + 365) =$$

$$= \frac{365}{2} \cdot 366 = 365 \cdot 183 = \$66,795$$