8.5 Applications of Matrices and Determinants

You will learn to

· Use Cramer's rule to solve a system by determinants.
· Determine the area of a triangle given three vertices on the coordinate plane.
· Write an equation of a line given two points.

Cramer's Rule

If \( ax + by = c \)  \( dx + ey = f \)

then \( x = \) \( y = \)

We can rewrite this as Cramer's rule:

\[
\begin{align*}
  x &= \frac{D_x}{D} \\
  y &= \frac{D_y}{D}
\end{align*}
\]
Example 1  Use Cramer's rule to solve this:

\[
\begin{align*}
5x - 2y &= 3 \\
6x + 4y &= -8
\end{align*}
\]

Cramer's rule can be used to solve a 3 x 3 system as well.

Example 2:

Set up the determinants for this system:

\[
\begin{align*}
-y + 2z &= 3 \\
4x + y &= 5 \\
x - 2z &= -6
\end{align*}
\]

\[
\begin{align*}
D &= \text{ } \\
D_x &= \text{ } \\
D_y &= \text{ } \\
D_z &= \text{ }
\end{align*}
\]
Area of a triangle with vertices \((x_1, y_1), (x_2, y_2), (x_3, y_3)\)

Test for collinearity

Example 2: Determine if these three points are collinear. If not, then find the area of the triangle which has them as the three vertices.

\[
\begin{align*}
A & \ (-3,4) & B & \ (2,0) & C & \ (5, -1)
\end{align*}
\]
Two point form of the equation of a line.

An equation of a line through the points \((x_1, y_1)\) and \((x_2, y_2)\) can be found using determinants.

Example 3: Write an equation of the line through the points (1, 5) and (0, -2)